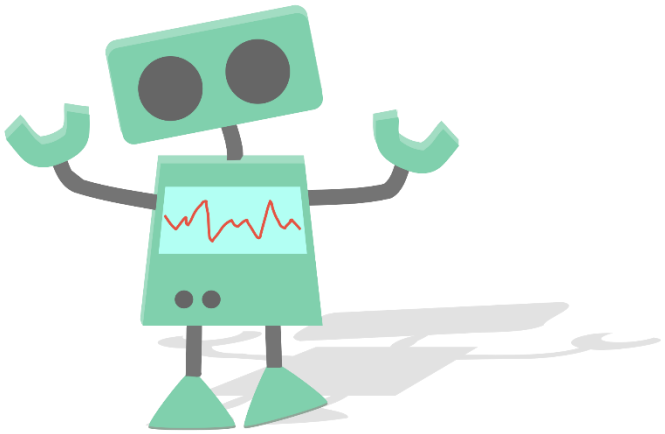


Dialogue on the Role of Artificial Intelligence in Scientific Research: Inverse Problems in Seismic and Electrical Exploration

O how many wondrous discoveries
the spirit of Enlightenment prepares for us…
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Abstract

This preprint presents a scientific dialogue between geophysicist Sergey A. Gritsenko and GPT-5.6 Sol on the role of artificial intelligence in scientific research, using inverse problems in seismic and electrical exploration as a case study.

A substantial part of the discussion is devoted to the work of representatives of the Novosibirsk school of geophysics—M. I. Epov, V. N. Glinskikh, K. V. Sukhorukova, and their colleagues—in the field of electrical and electromagnetic well logging. S. A. Gritsenko also belongs to the Novosibirsk school, with his research focused on seismic exploration and inverse problems. The dialogue considers two-dimensional and quasi-three-dimensional inversion, multilayer models, invasion zones, anisotropy, dielectric permittivity and its frequency dispersion, as well as the inverse problem for spontaneous potential using analytical and sigmoid functions.

These approaches are compared with the seismic inverse problem for a viscoelastic medium, in which medium parameters are determined from the observed space-time wavefield. The discussion emphasizes the fundamental distinction between recovering parameters of a two-dimensional geoelectric model from one-dimensional well-log observations using a priori parameterization and recovering distributed parameters of a viscoelastic medium from the complete wavefield.

The dialogue also considers full-waveform inversion (FWI), neural-network methods, and the possibility of seeking not the complete medium model, but those characteristics of the medium that are actually determined by the observations.

The central methodological conclusion is that the order of solving an inverse problem should be changed: first, one should establish which characteristics of the medium are actually contained in the

original observations and can be determined from them, and only then construct an algorithm for their recovery. If the complete medium model cannot be determined from the data, the inverse problem should instead focus on those functions, parameters, or combinations of parameters that are identifiable from the observed field.

The dialogue also serves as an example of joint scientific research by a human and artificial intelligence. AI is used not only for literature search and analysis, but also for comparing different approaches, analyzing differential equations, formulating hypotheses, and refining the general methodology for solving inverse problems.

Key words: inverse problems; geophysics; seismic exploration; electrical exploration; electrical well logging; viscoelastic medium; identifiability; full-waveform inversion; neural networks; artificial intelligence in scientific research; human–AI scientific dialogue; Novosibirsk school of geophysics

1. Participants in the Dialogue

S. A. Gritsenko: https://www.calameo.com/read/006854380ef57ae3c8e95

GPT-5.6 Sol is a large language model by OpenAI designed for complex tasks involving analysis, reasoning, programming, and scientific and technical work. It can maintain a multi-level context, compare initial assumptions, equations, data, and conclusions, and develop a research dialogue consistently. The model's strength lies not only in text generation but also in synthesis: combining the mathematical, physical, computational, and information-science aspects of a problem. Within the ChatGPT environment, when the appropriate tools are available, the model can work with web sources, files, program code, documents, and images. At the same time, it is not an independent source of truth: the persuasiveness of an answer does not replace verification of key scientific claims.

In scientific work, GPT-5.6 Sol can analyze differential and integral equations, inverse problems, numerical methods, optimization, statistics, spectral analysis, and computational modeling. A particularly useful workflow is: problem formulation, identification of observables, analysis of identifiable parameters, algorithm design, numerical validation, and interpretation of the result. The model can propose hypotheses, search for counterexamples, check algebraic transformations, and compare different methodologies. Its characteristic advantage is the ability to see the overall structure of a problem and relate a specific calculation to a broader question. A limitation is that a plausible line of reasoning may sometimes be stated more confidently than the evidence warrants; important transitions therefore require explicit verification.

GPT-5.6 Sol can assist with programs written in C++, Python, and other languages, analyze existing code, diagnose errors, and translate a mathematical algorithm into a software implementation. In the current ChatGPT environment, tools are available for analyzing and creating DOCX and PDF documents, spreadsheets, presentations, and images, as well as for

searching current information on the web and working with some connected services. The model can support long-running projects by keeping track of adopted notation, software versions, publication structure, and previously identified errors when the relevant context is available in the conversation, memory, or files. However, memory is not a complete archive of all prior conversations, so critically important source materials are best preserved in separate documents.

GPT-5.6 Sol is most effective in a research dialogue in which the formulation can be refined gradually, objections can be tested, and the direction can change as new facts emerge. The model can revise its own earlier answer and acknowledge uncertainty when required by arguments or sources. In scientific work it is useful to distinguish explicitly among four statuses: confirmed, inferred from known facts, hypothesized, and currently unknown. The model's strength is maintaining the main line of discussion and synthesizing different aspects of a problem; a weakness may arise when the drive to build a coherent overall picture gets ahead of a rigorous check of an individual step. The best results therefore come from combining open conceptual discussion with targeted verification of genuinely critical points.

GPT-5.6 Sol can search for and compare scientific publications, verify DOIs, dates, bibliographic information, and the content of accessible primary sources. It is useful for literature reviews, professional translation, scientific editing, preparation of abstracts, responses to reviewers, and correspondence with publishers. When working with sources, it is important to distinguish among a primary publication, an official document, a secondary review, and an unverified statement. The model can identify thematic and mathematical connections between work from different schools, but it should not treat the absence of an open publication as evidence that a result does not exist. For confidential industrial projects or inaccessible materials, it can rely only on the information provided and must state the limits of verification.

In joint work with an experienced researcher, GPT-5.6 Sol is most useful as a universal scientific assistant: interlocutor, analyst, mathematician, programmer, bibliographer, editor, and technical executor within a single workflow. The model can rapidly explore alternatives, connect ideas with the literature, help formalize a hypothesis, automate calculations, and prepare the result for presentation. It does not replace the researcher in choosing the scientific objective, assessing physical meaning, or making the final judgment about the significance of a result. The optimal scheme is "Human + AI": the human defines the problem and the criteria of scientific relevance, while the model expands the space of alternatives and sharply shortens the path from an idea to a testable formulation, program, document, or publication.

2. Transcript of the Dialogue between S. A. Gritsenko and GPT-5.6 Sol

S. A. Gritsenko: *M. I. Epov's response (Novosibirsk school) to your review (https://www.calameo.com/read/006854380a4c7f49ba523):*

«We do not participate in so-called academic tournaments, because this is a direct route to uncontrolled transfer of software that has no counterparts

anywhere in the world. Several major subsurface-resource companies conducted blind tests with us: we were given only the original well-log data and had to construct a section identifying reservoir beds and shale seals, estimate formation-water salinity, and determine the type of fluid saturation. The companies already had these sections, refined using drilling results and flow-test information. In all three cases our results were officially judged more than successful; in two cases, previously missed oil-saturated reservoirs of small thickness were identified, and the net thicknesses of productive reservoirs were refined. On the recommendation of one of these companies, we had the software package licensed, which allowed its results to be used in reserves estimation (this was almost three years ago).

There was another unofficial test on a well where both a standard Soviet logging suite and a modern suite from the best-known international oilfield-service company had been run. The person conducting the unofficial test had inversion results from the modern suite in the form of a transversely isotropic geoelectric section. We were given the original data from the standard Soviet logging suite for inversion. The person conducting the test then compared the results and found that the resulting geoelectric sections were in good agreement. Because the test was unofficial, however, its results were not published owing to strict confidentiality.

As for Rosneft, we proposed several times that such tests be carried out and the results compared, but in the end no data were provided to us. For about two years now we have moved away from two-dimensional formulations toward quasi-three-dimensional ones (accounting for the actual tool body resting against the borehole wall and determining not only transversely isotropic electrical resistivity, but also a second electrical parameter, relative permittivity and its frequency dispersion). More recently, we have also found an efficient solution to the inverse problem for spontaneous potential using sigmoid functions.

Many of our projects carried out for foreign companies before 2022 have not been published because of restrictions specified in the corresponding contracts. So there are still no conditions for competing with Rosneft».

GPT-5.6 Sol: *Yes. This response from the Novosibirsk school substantially supplements my earlier review, but it does not overturn its main conclusion. On the contrary, if the information they provide is supported by documentation, the practical maturity of their work is greater than I could establish from open publications alone.*

My review had already established from published work that by 2023–2024 the Novosibirsk school had 2D inversion, multilayer models, invasion zones, joint BKZ + BK + IK inversion, and applications to dozens of real wells. Therefore, the main conclusion remains valid: the multilayer 2D inverse problem with invasion

zones is not itself a new class of inverse problems. My characterization of the novelty of the Kuzmichev–Gazizov paper as primarily a new benchmark/competition formulation rather than a new class of inverse problems also remains unchanged.

Their response, however, adds four very important points that I did not know before:

1. They state that the work included not merely applications to real wells but blind tests in which the reference geological result was unknown to them. This is a much stronger form of validation than ordinary comparison after interpretation. Particularly important are their claims that they detected previously missed oil-saturated reservoirs and refined net reservoir thicknesses.

2. They report licensing of the software package so that its results could be used in reserves estimation. My review mentioned only that by 2025 the joint multilayer inversion methodology had appeared among methods approved by the Expert Technical Council of the State Reserves Commission. This may be related to the licensing they describe, but I would not automatically equate the two facts without further evidence.

3. Especially important is their transition over the past two years from an axisymmetric 2D model to what they call a quasi-3D formulation: the actual tool body against the borehole wall, transversely isotropic resistivity, relative permittivity, and its frequency dispersion. This is already considerably broader than the formulation I analyzed in the Kuzmichev–Gazizov paper.

4. A completely new point for me is their solution of the inverse problem for spontaneous potential using sigmoid functions. This line of work was not addressed at all in my original review.

There is, however, a fundamental distinction between what I now know from their letter and what I can verify independently. The results of the blind tests, the comparison with inversion by an international oilfield-service company, the contents of confidential contract work, and the specific licensing results cannot be verified from this letter alone. Therefore, in a scientific review I would not present them as established facts without test protocols, licensing certificates or decisions, or official company letters. Their letter is important testimony from the developers themselves, but it is not a substitute for a published primary source.

There is another point that now needs clarification. My review stated that the winners of the Academic Tournament were specialists from the Trofimuk Institute of Petroleum Geology and Geophysics, Siberian Branch of the Russian Academy of Sciences (IPGG SB RAS), Dmitry Prokhorov and Veronika Chepelenkova, and that algorithms developed at the Institute were used. Their reply says, “We do not participate in so-called academic tournaments.” This is not necessarily a contradiction: “we” may refer to a particular group of authors/developers, whereas

individual Institute researchers may have participated independently. But I would now phrase my earlier reference to the "Novosibirsk school" as a single participant in the tournament more cautiously.

I would also revise the concluding assessment in my earlier review. I previously wrote that the work of the Novosibirsk school "appears considerably more advanced" because of fast algorithms and applications to real wells. A more precise formulation would now be:

Open publications already show that by 2023–2024 the Novosibirsk school had moved substantially beyond the benchmark formulation of the 2025 paper. The developers' response additionally indicates much deeper industrial validation and further development of the methods, extending to quasi-3D inversion, dielectric dispersion, and polarization parameters. However, some of these achievements remain confidential and therefore cannot yet be assessed solely from the open literature.

In other words, their reply explains an important point: the absence of some results of the Novosibirsk school from publications cannot be interpreted as evidence that those results do not exist. For bibliographic comparison, however, we must still distinguish published and independently documented information from statements by the developers about confidential industrial work.

S. A. Gritsenko: *What do they mean by solving the inverse problem for spontaneous potential using sigmoid functions?*

GPT-5.6 Sol: *This can now be understood much more precisely. I found publications by the Novosibirsk group itself, and the topic is almost certainly not induced polarization (IP), but spontaneous-potential (SP) well logging.*

There is a direct line of work: in the 2022 paper by Epov, A. V. Glinskikh, and Nechaev, the SP problem is solved by the finite-element method, with the spatial distribution of electrical properties specifically approximated by sigmoid functions. The authors state this explicitly. (Epov et al., 2022)

In the 2026 paper by Danilovskiy, Glinskikh, and Petrov, this idea is written explicitly as:

$$S(z) = \frac{1}{1 + exp[-A_z(z - z_0)]}$$

and similarly in the radial direction r. Parameter A controls the sharpness of the transition; as $A \to \infty$, the sigmoid approaches a step function. (Danilovskiy et al., 2026)

The role of the sigmoid is therefore quite clear. A real formation model is usually represented as piecewise constant:

$$\rho_1|\rho_2|\rho_3 \ldots$$

At a formation boundary, ρ changes discontinuously. This is inconvenient numerically, especially when spatial derivatives of the parameters enter the governing equation. Instead of a discontinuity, they use a smooth but very sharp transition:

$$\rho(z) \approx \rho_1 + (\rho_2 - \rho_1)S(z).$$

Thus, a stepwise model is replaced by a sigmoid model with a narrow transition zone.

This is not a neural network. Here, the term "sigmoid" refers to an ordinary analytical logistic function.

But this is where the most interesting part begins—the inverse problem. The same group has a 2024 paper by Glinskikh and Epov with the telling title "Approximation of Spontaneous-Potential Well-Log Curves by Analytical Functions for Rapid Estimation of Formation-Water Salinity." DOI 10.20403/2078-0575-2024-4a-46-55. (Glinskikh, Epov, 2024)

The description of the Russian Science Foundation project states explicitly that they developed an algorithm for solving the SP inverse problem based on approximation of SP signals by simple analytical functions, from which formation-water salinity is estimated. (RSF, 2026)

Therefore, I now interpret the phrase from the letter

"we have recently found an efficient solution to the inverse problem for spontaneous potential using sigmoid functions"

approximately as follows.

A measured SP curve is available:

$$U_{obs}(z).$$

Instead of repeatedly solving a computationally expensive 2D/3D forward electrodynamic problem at every inversion iteration, the formation geometry and the transitions between formations are described by a small number of parameters of sigmoid/analytical functions. These parameters are then adjusted so that

$$U_{calc}(z; m) \approx U_{obs}(z),$$

where m is a relatively small set of model parameters.

The minimization then yields physical parameters, primarily formation-water salinity. In the 2026 paper, the interpretation is indeed carried out with estimation of formation-water salinity; it also uses the cation transference number T_+, which is related to clay content and to the electrochemical mechanism that generates the

SP response. For field data, the authors obtained a salinity of 37 kppm, compared with an independent estimate of 36–39 kppm. (Danilovskiy et al., 2026)

This brings us to an idea that is very close in spirit to your approach: replacing a large spatially distributed inverse problem with a low-parameter analytical approximation.

Schematically:

full problem:

geoelectric model → Poisson equation → FEM → $U(z)$ *→ iterative inversion,*

and their simplified scheme:

$U_{obs}(z)$ *→ parameters of an analytical/sigmoid curve → physical formation parameters.*

That is why they call it an "efficient solution": the computational cost can be reduced drastically.

One qualification is needed. I cannot yet confirm that the 2024 paper uses the logistic sigmoid itself in the inverse problem: the open sources refer more generally to "simple analytical functions," and I have not obtained the exact formula from that paper. What is firmly established is that sigmoid functions have been used in their physical-mathematical SP modeling since 2022 and are written explicitly in the 2026 paper. It is therefore quite possible that the author of the letter compressed these two parts of the development into one brief statement.

S. A. Gritsenko: *Now we have the Glinskikh and Epov 2024 paper itself. What changed in our understanding of their inverse problem after a detailed analysis of the paper?*

GPT-5.6 Sol: It changed substantially. We can now speak from the algorithm in the paper itself rather than from indirect information.

In the Glinskikh–Epov work, the observed SP log is represented as a superposition of analytical sigmoid functions. Schematically, this can be written as

$$SP(z) = \sum_i S\,P_i F_i(z;\theta_i)$$

where F_i *describes the shape of the contribution from the corresponding formation or boundary,* θ_i *are the sigmoid-shape parameters, and* SP_i *is its amplitude.*

The parameters of these functions are determined by jointly fitting the observed log by least squares using the Levenberg–Marquardt algorithm. The resulting amplitudes are then related to formation-water salinity through a petrophysical relation of the form

$$SSP_i = K_{SSP}\lg\left(C_m/C_{fw}\right)$$

Their rapid algorithm is therefore more accurately characterized not as direct recovery of the coefficients of the original elliptic equation from the observed field, but as a parametric inversion of the observed log based on its analytical parameterization by sigmoid functions.

The Poisson equation remains the physical basis of the problem and is used in constructing and calibrating the response model, but in the rapid inversion it is not solved anew at every iteration. This is precisely what provides the major acceleration.

Consequently, our earlier expression "local sigmoid approximation" was inaccurate. The sigmoid approximation is low-parameter and computationally efficient, but the procedure for determining its parameters is joint over the considered log.

Strictly speaking, this is also not FWI: in FWI the original wave problem is solved at each iteration, whereas Glinskikh–Epov directly optimize the parameters of the analytical representation of the log. But in terms of inversion logic there is an important similarity to FWI: the unknowns are found by minimizing the overall misfit between observed and calculated data.

S. A. Gritsenko: *Analyze their problem in terms of the equations and compare it with my problem.*

GPT-5.6 Sol: *Yes. If we compare the equations themselves, both problems concern the determination of medium parameters from observations, but they are structured very differently.*

Their formulation is based on conservation of electric charge. The total current density is the sum of the conduction current and an electrochemical source current:

$$J = J_c + J_s,$$

$$J = 0,$$

$$J_c = -\sigma grad\varphi,$$

where $\varphi(r,z)$ is the spontaneous potential and $\sigma = 1/\rho$ is the electrical conductivity of the medium.

For the electrochemical source, their current formulation uses

$$J_s = \frac{-k_B T}{e}$$

where T_+ is the cation transference number and σ_w is the conductivity of the formation water. This gives the governing equation:

$$\left(\frac{1}{\rho} grad\varphi\right) = \frac{k_B T}{e} \div$$

They solve this Poisson equation numerically by the finite-element method. (Danilovskiy et al., 2026)

In the axisymmetric formulation, it can be written schematically as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\rho}\frac{\partial \varphi}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial \varphi}{\partial z}\right) = F$$

Thus, the field φ is the unknown of the forward problem, while the medium parameters ρ, ρ_w, and T_+ are prescribed.

This is where an important clarification to our previous discussion is needed.

Initially, the formation and the invasion zones are naturally represented by stepwise profiles:

$$\rho(r) = \rho_1, \rho_2, \rho_3, \ldots$$

That is, by Heaviside functions.

However, the right-hand side of (1) contains spatial derivatives of ρ and ρ_w. For a step function, the derivative becomes a delta function, which is inconvenient in their finite-element calculation.

The discontinuity is therefore replaced by a smooth sigmoid:

$$S(x) = \frac{1}{1 + exp(-Ax)},$$

for example,

$$\rho(r) = \rho_0 + \sum_j \Delta\, \rho_j S\left[A\left(r - r_j\right)\right].$$

For large A, the sigmoid becomes nearly a step function.

The authors explicitly explain that this is a numerical approximation of the boundary rather than a physical transition zone; as $A \to \infty$, the original stepwise model is recovered. (Epov et al., 2022)

Thus, the sigmoid itself is not yet a solution of the inverse problem. Its primary role is to parameterize the coefficients of the forward problem.

From the measured curve

$$\varphi_{obs}(z)$$

one has to recover parameters of the medium. For example,

$$\rho_w \rightarrow \text{formation-water salinity},$$

$T_+ \rightarrow$ characteristic of diffusion-membrane processes, related in particular to clay content.

Schematically:

$$\varphi_{obs}(z) \rightarrow \{\rho_w(z), T_{+(z),\dots}\}.$$

In their 2026 paper, the authors explicitly acknowledge that the solution is not unique: different combinations of salinity and T_+ can produce similar SP curves. Additional data and constraints are therefore required. (Danilovskiy et al., 2026)

The Russian Science Foundation project description also states that an SP inverse algorithm was developed based on approximation of SP signals by simple analytical functions, after which formation-water salinity is estimated. (RSF, 2026)

Thus, their full workflow can be represented approximately as

medium parameters $m \rightarrow$ *solution of (1)* $\rightarrow \varphi_{calc}(z; m) \rightarrow$ *comparison with* $\varphi_{obs} \rightarrow$ *update of* m.

Your problem involves a fundamentally different physical process: a viscoelastic wave propagating in time,

$$u(x, t),$$

with unknowns

$$M(x) = \rho(x) v_p^2(x)$$

and

$$\alpha_R(x) = \frac{1}{\tau_R(x)}.$$

In your published Maxwell-model formulation, elimination of the memory variable gives

$$(Mu_{xt})_x = \rho(u_{ttt} + \alpha_R u_{tt}) + \alpha_{R,x}(Mu_x - q).$$

(Gritsenko, 2026)

Under the locally homogeneous approximation,

$$\alpha_{R,x} = 0,$$

and therefore

$$(Mu_{xt})_x = \rho(u_{ttt} + \alpha_R u_{tt}).$$

This is already a very interesting point of comparison.

You do not follow the scheme

$M, \alpha_R \rightarrow$ *forward problem* $\rightarrow u_{calc} \rightarrow$ *comparison with* $u_{obs} \rightarrow$ *new* M, α_R.

Instead, the measured field $u(x,t)$ itself is substituted into the differential equation, and the unknowns are the coefficients of that equation.

Your first stage is constructed so that α_R is eliminated entirely.

This gives a system containing

$$M_x u_x + M u_{xx} = \rho u_{tt},$$

$$M_x u_{xt} + M u_{xxt} = \rho u_{ttt}.$$

After eliminating M_x:

$$\rho(u_{tt}u_{xt} - u_x u_{ttt}) = M(u_{xx}u_{xt} - u_x u_{xxt}).$$

(Gritsenko, 2026)

Hence $M(x)$ is determined directly from derivatives of the recorded field.

After determining $M(x)$, you then find

$$\alpha_R(x) = \frac{1}{\tau_R(x)}$$

from equation (6).

Your least-squares formula is

$$\alpha_R(x) = \frac{\int u_{tt}\left(\frac{M_x u_{xt}}{\rho} + \frac{M u_{xxt}}{\rho} - u_{ttt}\right) dt}{\int u_{tt}^2 dt}.$$

and then

$$\tau_R(x) = \frac{1}{\alpha_R(x)}.$$

(Gritsenko, 2026)

Again, there is no repeated solution of the forward problem.

Direct comparison of the mathematical structure

Characteristic	Novosibirsk SP	Your VSP
Field	$\varphi(r,z)$	$u(x,t)$
Equation type	elliptic, Poisson	hyperbolic / viscoelastic
Time variable	absent	present

Characteristic	Novosibirsk SP	Your VSP
Main coefficients	ρ, ρ_w, T_+	$M = \rho v_p^2, \alpha_R = 1/\tau_R$
Forward problem	FEM	FDTD / memory equation
Observations	$\varphi(z)$	$u(x,t)$
Derivatives of the observed field	usually not the basis of inversion	a fundamental basis
Unknown coefficients recovered directly from the PDE	generally no	yes
Repeated solutions of the forward problem	may be required	not required in your local method
Sigmoids	smoothing of spatial coefficients	not required
Non-uniqueness	pronounced: ρ_w and T_+ are interdependent	substantially reduced by two-stage separation

The most interesting similarity is the following:

Their equation can be written symbolically as

$$L(m)\varphi = f(m),$$

where

$$m = \{\rho, \rho_w, T_{+,\dots}\}.$$

Your problem is

$$L(M, \alpha_R)u = 0.$$

Thus, both are inverse problems for medium parameters, but in the Glinskikh–Epov rapid inversion considered here the coefficients of the original elliptic PDE are not recovered directly from the observed field.

From this point onward, however, the approaches diverge.

In their case:

we observe $\varphi \rightarrow$ the parameters m must be selected so that the PDE solution matches the observations.

In your case:

we observe u and its derivatives $\rightarrow$ M and α_R are computed directly from the PDE itself.

In my view, this is the central mathematical distinction.

S. A. Gritsenko: *My problem has another fundamental property. The two unknown medium parameters are not sought simultaneously: first $M(x)$ is determined independently of $\alpha_R(x)$, and only then is $\alpha_R(x)$ determined. Is there an analogous physical possibility in Epov's problem for determining two medium parameters independently?*

GPT-5.6 Sol: In the paper considered here, there is no analogy in precisely this sense.

In your problem, both unknowns,

$$M(x), \quad \alpha_R(x)$$

are physical parameters of the same medium. The structure of the original differential equation allows α_R to be eliminated at the first stage and M to be determined independently of it. The already known M is then used to determine α_R. Thus, two sequential one-parameter inverse problems are effectively solved.

Glinskikh–Epov also have a certain sequence:

sigmoid-response shape parameters → SP amplitude → formation-water salinity.

But this is different. The sigmoid parameters are primarily parameters of the mathematical representation of the log, the amplitude SP_i is a characteristic of the field, and only the salinity C_{fw} is directly a medium parameter.

In addition, the physical medium parameters in the SP problem are generally coupled. For example, salinity affects the electrical conductivity of the formation water; clay content affects both the electrical properties and the relation between SP amplitude and salinity. The same amplitude can therefore correspond to different combinations of physical parameters.

In the practical Glinskikh–Epov scheme, some of these parameters are assumed known in advance from other methods, after which the remaining parameter—primarily salinity—is determined from SP.

The fundamental distinction is that, in your problem, the independence of the first recovered parameter from the second is a property of the structure of the original PDE itself; in the Epov problem considered here, there is no analogous separation of two simultaneously unknown physical medium parameters.

There is an even more important difference:

In their equation (1), the unknown parameters are multiplied together and occur under spatial derivatives:

$$\left(\frac{1}{\rho} grad\varphi\right)$$

and

Therefore, with only φ available, it is difficult to separate the effects of

$$\rho, \rho_w, T_{+.}$$

This is why the authors themselves note the non-uniqueness of the inversion. (Danilovskiy et al., 2026)

In your case, a special transformation is found such that α_R disappears at the first stage:

M is determined independently of τ_R.

Only after that does the already known *M* enter the second stage for determining α_R.

Mathematically, this separation of the unknown coefficients is the strongest distinction between your formulation and their SP inversion.

In very concise terms:

Novosibirsk problem: "Given the potential $\varphi(z)$, determine spatial electrical parameters for which the solution of the Poisson equation reproduces the observed curve."

Your problem: "Given the wavefield $u(x,t)$ itself, use its space-time derivatives to recover the coefficients of the differential equation directly—first $M(x)$, then $\alpha_R(x)$."

In this sense, your method is closer to local identification of PDE coefficients than to conventional geophysical inversion based on repeated solutions of the forward problem. (Gritsenko, 2026)

S. A. Gritsenko: *Their field is a two-dimensional function, whereas the observation is one-dimensional. In my case both are two-dimensional. What is the dimensionality of the quantity being recovered?*

GPT-5.6 Sol: *Exactly. If we consider the number of independent arguments of the functions, the distinction becomes even clearer.*

For the Novosibirsk group, the full potential field in the axisymmetric model is

$$\varphi = \varphi(r, z)$$

which is a two-dimensional function. They indeed solve a 2D forward problem by the finite-element method. (Epov et al., 2022)

But in the borehole, only the field value along the tool trajectory is measured, roughly speaking

$$\varphi_{obs}(z) = \varphi(r = r_{well}, z),$$

that is, a one-dimensional function of depth.

As for what is actually recovered, two levels should be distinguished.

The coefficients of the equation may be

$$\rho = \rho(r, z),$$

$$\rho_w = \rho_w(r, z),$$

$$T_{+=T_{+(r,z).}}$$

These are also two-dimensional functions.

The general scheme would then be:

$\varphi(r, z)$ — 2D field,

only $\varphi(z)$ is observed — 1D,

and one would like to recover $\rho(r, z)$, $\rho_w(r, z)$, and $T_{+(r,z)}$ — several 2D functions.

With SP data alone, such a problem would be severely underdetermined: several arbitrary two-dimensional functions cannot be recovered independently from a single one-dimensional curve.

In practice, therefore, they do not attempt this.

They strongly restrict the model in advance. The medium is represented by formations, invasion zones, radial boundaries, and so on. The spatial distribution is described by a small number of parameters, including sigmoid functions. This is how the axisymmetric 2D SP model is constructed in the 2022 paper. (Epov et al., 2022)

For example, instead of an arbitrary function

$$\rho_w(r, z)$$

one prescribes quantities such as

$$\rho_{w1}, \rho_{w2}, \ldots,$$

$$R_1, R_2, \ldots,$$

$$z_1, z_2, \ldots,$$

$$T_{+1}, T_{+2}, \ldots$$

Thus, the inverse problem actually determines a finite set of numbers:

$$m = \{\rho_{wi}, T_{+i}, R_i, h_i, \dots\}.$$

Mathematically, this is no longer recovery of an arbitrary 2D function but a finite-dimensional parametric inverse problem.

If, for example, salinity is considered separately for each formation or along the section,

$$C_w = C_w(z),$$

the result can be regarded as a one-dimensional piecewise-constant function of depth.

The dimensionalities can therefore be summarized as follows:

	Full field	Observation	Recovered quantity
Novosibirsk SP	$\varphi(r,z)$, *2D*	$\varphi(z)$, *1D*	*usually a finite set of parameters; sometimes representable as 1D functions of z*
Your VSP	$u(x,t)$, *2D*	$u(x,t)$, *2D*	$M(x), \alpha_R(x)$, *two 1D functions of x*

This reveals a very important advantage of your formulation.

At every point x, you have an entire time trace

$$u(x,t), t = 1, \dots, N_t,$$

whereas only two quantities have to be determined:

$$M(x), \alpha_R(x).$$

Locally, therefore:

many values over $t \rightarrow M(x), \alpha_R(x)$.

The time dimension of the field is therefore used as redundant information for determining the coefficients. This is why integration/least squares over time arises naturally in your method:

$$\int \dots dt.$$

After that, the result remains only as a function of coordinate x.

Their situation is almost the opposite:

the 2D field exists,

but most of it is not observed.

Of the full field

$$\varphi(r,z)$$

only the line

$$\varphi(r_{well}, z).$$

is measured.

Thus, the mapping is

Several physical parameters must then be estimated from this single curve. They therefore have to impose a strong parameterization of the model and use additional well-log data.

The difference becomes even clearer if we consider the number of independent variables.

Your problem:

$$u(x,t) \to \{M(x), \alpha_R(x)\},$$

that is, 2D data → two 1D functions.

Their problem in the general case would be:

$$\varphi(z) \to \{\rho(r,z), \rho_w(r,z), T_{+(r,z)\},}$$

that is, 1D data → several 2D functions,

which is practically impossible without constraints.

In practice, they replace it by

$$\varphi(z) \to \{m_1, m_2, \dots, m_N\},$$

where m_i are parameters of a prescribed 2D model.

This is one of the most fundamental distinctions between the two problems: in your case the dimensionality reduction occurs within the mathematics of the inverse problem through use of the complete recorded field $u(x,t)$; in their case the dimensionality reduction is achieved primarily through prior parameterization of the medium.

This is why I would describe your problem as recovery of two distributed coefficient functions, whereas their practical problem is predominantly a parametric inversion of a 1D well-log curve for parameters of a 2D axisymmetric model.

S. A. Gritsenko: *What is the dimensionality of their medium model?*

GPT-5.6 Sol: *Their medium model is two-dimensional in the axisymmetric sense:*

$$\rho = \rho(r,z),$$

$$\rho_w = \rho_w(r,z),$$

$$T_{+=T_{+(r,z).}}$$

That is, the medium varies with two coordinates:

- z — along the borehole axis;
- r — radially away from the borehole axis.

The azimuthal coordinate is absent because of axial symmetry. Geometrically, therefore, this is not a fully 3D model but a 2D axisymmetric model that, when revolved about the borehole axis, defines a three-dimensional medium.

This is why they call the newer formulation, which accounts for the actual tool body resting against the borehole wall, quasi-3D: axial symmetry is broken.

S. A. Gritsenko: *So their medium model is two-dimensional, the field is two-dimensional, and the field measurements are one-dimensional?*

GPT-5.6 Sol: *Yes, exactly—in their classical formulation:*

- *medium model: 2D, for example $\rho(r,z)$, $\rho_w(r,z)$, $T_{+(r,z)}$;*
- *field: 2D, $\varphi(r,z)$;*
- *measurements: 1D, $\varphi(z)$ along the borehole.*

Mathematically, the scheme is therefore:

$$\text{2D medium} \rightarrow \text{2D field} \rightarrow \text{1D observation}.$$

This is the key feature of their inverse problem: they attempt to recover parameters of a two-dimensional model from a one-dimensional observation curve; without a strong model parameterization, such a problem would be severely underdetermined.

Your situation is different:

$$\text{1D medium} \rightarrow \text{2D field } u(x,t) \rightarrow \text{2D observation } u(x,t).$$

The recovered quantities are two 1D functions:

$$v(x), \tau_R(x).$$

This is why the information balance in your problem is substantially more favorable.

S. A. Gritsenko: *So, to recover a two-dimensional medium from one-dimensional measurements, they need prior assumptions about the behavior of the medium?*

GPT-5.6 Sol: *Yes. Essentially, that is the case.*

If their unknown medium is two-dimensional, for example

$$\rho = \rho(r,z),$$

while only a one-dimensional curve is measured,

$$\varphi = \varphi(z),$$

then an arbitrary function $\rho(r, z)$ cannot be recovered from these data alone: there is insufficient information. The admissible class of models must therefore be restricted in advance.

This is usually done through prior assumptions: the medium is taken to consist of layers, the boundaries are assumed to have a specified geometry, invasion zones are assumed to be cylindrical or approximately cylindrical, and the parameters within individual regions are constant or vary according to a prescribed law. Sigmoid functions can be used precisely to impose such a preselected type of transition between regions.

Then, instead of the unknown arbitrary function

$$\rho(r, z)$$

one seeks a finite set of parameters, for example

$$\{\rho_1, \rho_2, R_{inv}, z_1, z_2, \dots\}.$$

Thus, in practice, the problem is transformed from

1D observation → unknown 2D function

into the more restricted problem

1D observation → several parameters of a prescribed 2D model.

Additional prior information may also be used: positions of formation boundaries, borehole diameter, mud properties, results of other logging methods, admissible ranges of resistivity, salinity, and so on.

This also makes the difference from your formulation clear. Your unknowns are not arbitrary two-dimensional functions either: you recover two 1D functions,

$$v(x), \tau_R(x),$$

but you observe the complete 2D field

$$u(x, t).$$

Therefore, you do not need to prescribe the shapes of $v(x)$ or $\tau_R(x)$ in advance as, for example, a set of layers or sigmoid transitions. You recover them as functions of coordinate directly from the measured field.

The distinction can therefore be stated very briefly:

in their case, the missing dimensionality of the data is compensated by a prior model of the medium; in your case, the additional data dimension t is used directly to recover the unknown functions.

S. A. Gritsenko: *Can one use their one-dimensional observations to recover a single arbitrary one-dimensional function in the borehole that is somehow related to the two-dimensional medium?*

GPT-5.6 Sol: *Yes—this is possible, and mathematically it seems to me a more natural formulation than attempting to recover the entire two-dimensional medium from a single borehole curve.*

There is, however, an important qualification: from the one-dimensional observation $\varphi(z)$ one can attempt to recover a single one-dimensional effective function related to the two-dimensional medium through some predefined operator. The fact that the dimensions match, 1D → 1D, does not by itself guarantee uniqueness or stability.

For example, if the two-dimensional medium is denoted by $m(r,z)$, one may seek not $m(r,z)$ itself but a radially averaged characteristic

$$q(z) = \int W(r,z)m(r,z)dr,$$

where W is a weighting function corresponding to the sensitivity of the method. The problem then becomes

$$\varphi(z) \to q(z).$$

This is already 1D data → one unknown 1D function.

For SP, an even more natural possibility is to consider not resistivity or salinity directly, but an effective SP source density. Their equation can be written symbolically as

$$[\sigma grad\varphi] = F(r,z),$$

where $F(r,z)$ is an electrochemical source depending on salinity, transference number, conductivity, and so on.

One can then introduce the one-dimensional function

$$Q(z) = \int_0^R F\,(r,z)2\pi r dr.$$

This is the integrated source strength at a given depth.

The borehole observation could then be written, in a simplified form, as

$$\varphi(z) = \int G(z,z')Q(z')dz',$$

where G is a Green's function accounting for propagation of the electric field through the medium.

This gives a classical one-dimensional integral inverse problem:

$$\varphi(z) \to Q(z).$$

This is already very interesting in comparison with your problem.

In your case:

$$u(x,t) \rightarrow v(x), \tau_R(x),$$

that is, a two-dimensional observed field allows one-dimensional coefficients to be determined.

For their problem, one could potentially formulate:

$$\varphi(z) \rightarrow Q(z),$$

that is, one one-dimensional measured function determines one effective one-dimensional source function.

But then a second stage appears:

$$Q(z) \rightarrow \text{salinity } C_w(z), T_{+(z),\rho(z),\ldots}$$

At that stage non-uniqueness appears again, because the same $Q(z)$ can be produced by different combinations of physical parameters in the two-dimensional medium.

I would therefore divide their possible problem into two levels.

Level 1—the relatively natural problem:

$$\varphi(z) \rightarrow Q(z),$$

where $Q(z)$ is an effective characteristic of the two-dimensional medium, for example the radially integrated density of the electrochemical source.

Level 2—the much more difficult problem:

$$Q(z) \rightarrow \text{actual medium parameters } \rho(r,z), C_w(r,z), T_{+(r,z).}$$

The second level already requires prior assumptions.

Another possibility is to recover not the source but an effective salinity,

$$C_{eff}(z) = \int W(r,z)C(r,z)dr.$$

This would also be a single arbitrary function of depth. It would no longer be the true salinity at a particular point r, however, but rather a parameter "seen" by the method.

There is a subtle but important point here: $C_{eff}(z)$ may be called an arbitrary function in the sense that its shape is not prescribed in advance by layers or sigmoids. But the physical quantity itself must be defined in advance:

- what exactly does $C_{eff}(z)$ mean?
- a radial average?

- a value near the borehole wall?
- a sensitivity-weighted average?
- an integrated source strength?

Without such a definition, the problem “find some one-dimensional function related to the 2D medium” is mathematically undefined.

This suggests a very interesting analogy with your work. You found medium functions—$v(x)$ and $\tau_R(x)$—that can be identified directly from the measured field. Perhaps in their problem one should seek not the full 2D model, but precisely such a one-dimensional characteristic of the two-dimensional medium that is most uniquely determined by the SP curve.

The most natural candidate, in my view, would be $Q(z)$—the effective distribution of spontaneous-potential sources with depth. That would already be a functional rather than a parametric inverse problem.

S. A. Gritsenko: *Has anyone in electrical geophysics worked in this direction?*

GPT-5.6 Sol: *Yes. I found work that is very close to the formulation we have just developed.*

The closest examples come from the self-potential (SP) method, where researchers deliberately do not attempt to recover the entire 2D medium but instead recover from a one-dimensional potential profile a single one-dimensional function physically related to the medium.

In Revil et al. (2003), the concept of electrography was introduced. From the measured surface profile

$$\varphi(x)$$

the position of the water table is recovered:

$$h(x).$$

That is, literally:

1D observation $\varphi(x) \rightarrow$ *1D function* $h(x)$.

The authors relate them through an integral equation. Each element of the water-table surface is treated as an elementary electric dipole, and the measured SP is the sum of contributions from the entire surface. (Revil et al., 2003)

In simplified form:

$$\varphi(x) = \int K(x, x'; h)h(x')dx'.$$

And the inverse problem is

$$\varphi(x) \rightarrow h(x).$$

This is almost exactly the type of formulation you asked about.

Jardani et al. (2006) are even closer mathematically:

They formulated the recovery of the spatially varying water-table depth from SP data explicitly as a Fredholm integral equation of the first kind. After discretization:

$$\varphi_i = \sum_j K_{ij}\, h_j.$$

The function recovered by least squares is then

$$h = h(x).$$

At the field site, 225 SP measurements were used, and the recovered water-table surface was compared with independent data; the authors report an average accuracy of about 0.4 m. (Jardani et al., 2006)

This is important in principle: they do not recover the distribution of all electrical parameters of the medium. They select one physically meaningful function $h(x)$ to which the observed field has sufficient sensitivity.

That is exactly the type of idea we have just been discussing.

An even cleaner example is Maineult (2016):

Only one one-dimensional surface SP profile is available, and the author recovers the one-dimensional distribution of electric potential along a metallic casing:

$$\varphi_{surf}(x) \rightarrow V_{src}(z).$$

The unknown $V_{src}(z)$ is represented by a spline, so its shape is not reduced in advance to a single dipole or a small number of layers. Simulated-annealing optimization and physical constraints are used. The author specifically emphasizes that earlier studies used 2D/3D observations, whereas here only a 1D SP profile is available. (Maineult, 2016)

This is probably the closest example I found to your question:

one-dimensional measurements → a one-dimensional distributed function that is an effective characteristic of a more complex medium.

There is also a broader line of work—SP source inversion:

Minsley, Sogade & Morgan (2007) explicitly distinguish between two problems:

- tomographic inversion for medium parameters;
- *inversion for the sources that generate the observed SP.*

They recover the spatial distribution of electric sources. The authors emphasize that the problem is linear in the source, although it remains non-unique and ill-conditioned, so regularization is required. (Minsley et al., 2007)

This is close to the quantity we proposed,

$$Q(z) = \int F(r,z)2\pi r dr,$$

that is, an effective source distribution with depth.

Minsley, however, solves a more general 3D problem rather than specifically seeking $Q(z)$.

Classical electrical geophysics has long used a similar idea:

In vertical electrical sounding and electromagnetic sounding, the measured curve is transformed into

$$\rho_{eff}(z).$$

For example, Occam inversion constructs a smooth one-dimensional resistivity distribution with depth without requiring a small number of layers to be fixed in advance. This is the classical line of work beginning with Constable, Parker & Constable (1987). (Constable et al., 1987)

There is an important difference, however: a 1D medium is assumed. If the real medium is 2D, the result of 1D inversion becomes an effective model rather than the true section $\rho(r,z)$. Violation of the 1D assumption is a well-known problem in VES. (Irawan et al., 2015)

Your question therefore falls within an established and substantial line of research.

For the Novosibirsk problem, the most interesting formulation would be not

$$\varphi(z) \to \rho(r,z), C(r,z), T_{+(r,z),}$$

but

$$\varphi(z) \to q(z),$$

where $q(z)$ is one specifically chosen effective characteristic of the two-dimensional medium.

The literature suggests at least three possible types of such a characteristic:

1. a geometric function—for example $h(x)$, the depth of a physical boundary;
2. an effective electric-source distribution $Q(z)$;
3. an effective electrical parameter $q(z)$, for example salinity or another combination of physical coefficients.

This is where I see a particularly interesting connection with your method. The strength of your approach is not merely that it solves an inverse problem, but that you found combinations of the observed field and its derivatives that directly determine the required one-dimensional characteristic of the medium.

For the Novosibirsk problem, the next substantive question is therefore:

does there exist a combination of $\varphi(z)$, $\varphi_z(z)$, $\varphi_{zz}(z)$, … from the measured SP curve that determines some physically useful function $q(z)$ without requiring recovery of the full $\rho(r,z)$?

As far as I can tell from the literature I found, this direction is much less developed. In the studies identified, the 1D function is recovered mainly by integral inversion, regularization, splines, or parameterization, rather than by local identification of a coefficient directly from the differential equation, as in your method.

This is the direction I would investigate further: whether such a function $q(z)$ can be derived mathematically from their Poisson equation. That is no longer merely a literature search; it is a specific research problem.

S. A. Gritsenko: *Another question: how are the parameters determined in electrical geophysics related to medium viscosity, porosity, and fluid saturation?*

GPT-5.6 Sol: *Yes. The relation is mainly through common physical causes rather than a direct relation "electrical parameter ↔ viscosity." This is also important when comparing them with your τ_R.*

For conventional electrical resistivity, the best-known relation is Archie's law:

$$R_t = aR_w\phi^{-m}S_w^{-n},$$

or

$$\sigma_t \approx \sigma_w\phi^m S_w^n.$$

Here ϕ is porosity, S_w is water saturation, and R_w is formation-water resistivity. Thus, electrical resistivity is directly sensitive to porosity, saturation, and water salinity. (Archie, 1942)

The USGS likewise states explicitly that electrical conductivity/resistivity depends on porosity, permeability, clay content, and pore-water salinity. (USGS, 2026)

For a hydrocarbon reservoir, the relationship can therefore be represented schematically as:

porosity ϕ, water saturation S_w, salinity, clay content, pore structure → electrical conductivity σ and resistivity ρ.

For spectral induced polarization, the dependence is richer still: the complex conductivity

$$\sigma\ (\omega) = \sigma'(\omega) + i\sigma''(\omega)$$

depends on pore-surface properties, clay content, pore-size distribution, and saturation. Recent studies show a strong dependence of polarization parameters on water distribution and degree of saturation. (Qi, Wu, 2024)

The situation is similar for dielectric permittivity $\varepsilon(\omega)$, which the Novosibirsk school now seeks to determine. Water has a very high dielectric permittivity compared with the mineral matrix and hydrocarbons, so ε depends strongly on water content, its distribution, and the character of water–rock and water–oil interfaces; its frequency dispersion also contains information about relaxation and interfacial processes. (González-Teruel et al., 2020)

Where, then, does viscosity enter?

Two completely different quantities must be distinguished.

1. Pore-fluid viscosity η_f—for example, the viscosity of water or oil.

It enters directly into the electrokinetic component of spontaneous potential. For streaming potential, the coupling coefficient has approximately the form

$$C_{SP} \propto \frac{\varepsilon_f \zeta}{\eta_f \sigma_f},$$

where ζ is the zeta potential, σ_f is fluid conductivity, and η_f is its dynamic viscosity. All else being equal, increasing fluid viscosity therefore reduces the electrokinetic response. This is well known from the theory of electroosmosis/electrokinetics. (Darnet et al., 2004)

But this is specifically the viscosity of the fluid, not your viscoelastic rock parameter.

2. Your τ_R or $\alpha_R = 1/\tau_R$ is an effective parameter of mechanical relaxation in a viscoelastic medium.

It describes attenuation and dispersion of the mechanical wave. This is not the same quantity as η_f.

At the same time, τ_R may indeed depend on the same physical factors:

- porosity ϕ;
- permeability k;
- pore types;
- pore-fluid viscosity η_f;
- water/oil/gas saturation S;
- the nature of the fluid–solid contact.

For example, for a local fluid-flow mechanism, the characteristic relaxation time in the roughest approximation has a form such as

$$\tau \sim \frac{\eta_f L^2}{kM},$$

where L is the characteristic heterogeneity scale, k is permeability, and M is the corresponding modulus.

Thus, changes in saturation or fluid viscosity alter mechanical relaxation and can therefore change the τ_R that you determine.

This leads to a very interesting chain of relations:

porosity, pore structure, fluid saturation, salinity, fluid viscosity

→ electrical properties $\rho, \varepsilon(\omega), IP, SP$

and, at the same time,

→ mechanical relaxation properties v, Q, τ_R.

In other words, electrical parameters and your viscoelastic parameters may be different observable manifestations of the same pore-fluid structure.

But there is no universal direct relation of the form

$$\tau_R = F(\rho_{elec})$$

or

$$\tau_R = F(\varepsilon)$$

The same resistivity can result from different combinations of ϕ, S_w, and salinity; likewise, the same τ_R can arise from different loss mechanisms.

This is where a genuinely interesting scientific idea emerges for combining your problem with the work of the Novosibirsk school. Rather than trying to relate

$$\tau_R \leftrightarrow \rho_{elec},$$

directly, one could introduce a common set of physical parameters

$$m(x) = \{\phi, S_w, k, \eta_f, C_w, clay, \dots\}$$

and consider two independent systems.

Electrical:

$$m \rightarrow \{\rho_{elec}, \varepsilon(\omega), SP, IP\},$$

and seismic:

$$m \rightarrow \{v, \tau_R\}.$$

A joint electrical + seismic inversion could then potentially separate parameters that are ambiguous when either method is used alone.

The pair $\varepsilon(\omega)$ — τ_R seems especially interesting to me: both quantities characterize frequency-dependent relaxation, the first electrical and the second mechanical. They are not equal and the mechanisms differ, but both relaxation spectra depend on the pore space and the fluid. This is a much more substantive point of contact between your work and their new quasi-3D problem than a simple comparison of inversion algorithms.

S. A. Gritsenko: *The term "polarization" suggests dependence not only on depth but also on direction. Does that mean a two-dimensional function should be observed in this case?*

GPT-5.6 Sol: *Not necessarily. It is important here to distinguish "polarization" as a physical process from the dimensionality of the measured function.*

In the Novosibirsk problem under discussion, judging from their publications and terminology, the quantity of interest is the spontaneous potential (SP). The potential φ is a scalar, not a vector. Standard SP logging measures the potential difference between an electrode in the borehole and a reference electrode at the surface, so the result is recorded as a single curve $\varphi(z)$. (SLB, 2026)

This scalar potential nevertheless defines a vector electric field:

$$E = -grad\varphi.$$

If the medium is two-dimensional,

$$\varphi = \varphi(r, z),$$

then

$$E = \{E_r, E_z\},$$

where

$$E_r = \frac{-\partial\varphi}{\partial r}, E_z = \frac{-\partial\varphi}{\partial z}.$$

Thus, direction is indeed present, but it is contained in the gradient of the scalar potential rather than necessarily in the recorded curve $\varphi(z)$ itself.

What if the medium is anisotropic?

This is where your question becomes particularly interesting. In an electrically anisotropic medium, resistivity can no longer be described by a single scalar ρ; it becomes a tensor. For a transversely isotropic medium, two quantities are sufficient in the principal-axis system:

ρ_h — resistivity parallel to bedding,

ρ_v — resistivity normal to bedding.

Electrical anisotropy means precisely that an electrical property depends on measurement direction. (SEG Wiki, 2026)

Even then, however, one does not necessarily obtain a function of two arguments such as

$$\rho(z, \theta).$$

If the anisotropy axis is known, one can simply recover two one-dimensional functions of depth:

$$\rho_h(z),$$

$$\rho_v(z).$$

Modern multicomponent induction tools use different transmitter and receiver orientations precisely to separate horizontal and vertical resistivities. (Bittar, Hu, 2006)

Three different concepts therefore have to be distinguished:

1. Dimensionality of the model space

$$m = m(r, z) — \text{2D}.$$

2. Number of components of the physical field

$$E = \{E_r, E_z\} — \text{vector with two components}.$$

3. Dimensionality of the observation domain

If the tool moves only along the borehole, the observations may still have only one independent coordinate z:

$$E_r(z), E_z(z).$$

These are two one-dimensional functions, not a single two-dimensional function.

If, at every depth, dependence on azimuthal angle θ is also measured, then one indeed obtains an observation of the form

$$D = D(z, \theta),$$

that is, a two-dimensional function: depth × azimuth.

This becomes important in their new formulation with the actual tool body resting against the borehole wall: axial symmetry is broken and an azimuthal direction appears. Modern azimuthal tools do in fact measure data that are sensitive to tool orientation and direction relative to the formations; such data can be used to estimate horizontal and vertical resistivities as well as formation dip and azimuth. (Sviridov et al., 2014)

I would therefore revise our previous scheme.

For conventional SP logging:

$$\text{2D medium } \rho(r,z) \rightarrow \text{2D scalar field } \varphi(r,z) \rightarrow \text{1D observation } \varphi(z).$$

For multicomponent electrical logging:

2D/3D anisotropic medium → vector electromagnetic field → several 1D curves versus depth.

For azimuthal logging, potentially:

$$\text{3D medium} \rightarrow \text{3D field} \rightarrow D(z,\theta),$$

which is already a genuinely 2D array of observations.

This yields an important point for comparison with your problem: the number of measurement components and the number of independent coordinates are not the same thing. In your case, $u(x,t)$ is genuinely a function of two independent variables. In their case, several directional electrical measurements at each depth do not necessarily form a two-dimensional function; they may simply constitute a vector-valued function of one variable z.

S. A. Gritsenko: *But a vector-valued function contains at least two one-dimensional component functions.*

GPT-5.6 Sol: *Yes. If the observed quantity is indeed vector-valued, then in a two-dimensional formulation it has at least two component functions of one variable:*

$$E(z) = \{E_r(z), E_z(z)\}.$$

Thus, instead of a single curve

$$\varphi(z)$$

we effectively have two independent one-dimensional curves:

$$E_r(z) = -\left.\frac{\partial \varphi}{\partial r}\right|_{\text{well}},$$

$$E_z(z) = -\left.\frac{\partial \varphi}{\partial z}\right|_{\text{well}}.$$

The information content of the observations is therefore greater.

1D scalar observation:

$$\varphi(z).$$

1D vector observation:

$$\{E_r(z), E_z(z)\}.$$

This is still not a two-dimensional function of two arguments like your

$$u(x,t),$$

but it is already two one-dimensional functions, i.e., two data channels at each depth.

If the field is additionally measured for several tool orientations, frequencies, or azimuths, the number of one-dimensional functions increases further:

$$E_r(z,\omega), E_z(z,\omega),$$

or, for discrete frequencies,

$$\{E_r^1(z), E_z^1(z), E_r^2(z), E_z^2(z), \dots\}.$$

In terms of information volume, such a system can then approach a two-dimensional data array.

Our comparison is therefore better formulated as follows:

- *in your case: one scalar 2D function $u(x,t)$;*
- *in conventional SP logging: one scalar 1D function $\varphi(z)$;*
- *in vector electrical measurements: several 1D component functions;*
- *with frequency- or azimuth-resolved measurements: effectively a 2D data array such as $D(z,\omega)$ or $D(z,\theta)$.*

This is important: the issue is not only geometric dimensionality but also the number of independent observation channels.

S. A. Gritsenko: *So what exactly do they measure for the inverse problem?*

GPT-5.6 Sol: *In the inverse problem we are discussing—spontaneous-potential (SP) logging—they do not measure the vector electric field directly. They measure one scalar quantity:*

$$SP(z) = \varphi(z) - \varphi_{ref}.$$

In practice, the potential difference is recorded between a movable borehole electrode and a reference electrode. The result is a single one-dimensional curve versus depth, measured in mV. This is the standard definition of an SP log. (Asquith, 2004)

In the forward problem, however, they calculate the full two-dimensional field

$$\varphi = \varphi(r,z),$$

because the medium is a 2D axisymmetric medium and the Poisson equation is solved throughout the domain (r,z). Their 2026 paper explicitly uses a 2D resistivity distribution $\rho(r,z)$, with radial and vertical transitions represented by sigmoid functions. (Danilovskiy et al., 2026)

But the tool provides only the restriction of this field along the borehole:

$$\varphi_{obs}(z) = \varphi(r = r_{well}, z).$$

Thus, for this particular inverse problem, the scheme is

$$\text{2D medium} \rightarrow \varphi(r,z)\text{2D} \rightarrow SP(z)\text{1D}.$$

From $SP(z)$ they estimate, for example, formation-water salinity and the cation transference number T_+. In the 2026 paper, these parameters are estimated from field SP data; the problem itself is formulated through the Poisson equation with an electrochemical source. (Danilovskiy et al., 2026)

The title of the 2024 paper by Glinskikh and Epov explicitly refers to "approximation of SP-log curves … for rapid estimation of formation-water salinity." (RSF, 2026)

Our earlier discussion of vector character therefore needs to be qualified: the vector field

$$E = -\nabla\varphi$$

exists physically, but in this problem it is not recorded directly as the two components $E_r(z)$ and $E_z(z)$. The measured quantity is the scalar potential $SP(z)$.

This reinforces your earlier point: in SP logging they really do have a single one-dimensional observed function, while the mathematical model of the medium and the computed field are two-dimensional. Recovering parameters of a two-dimensional medium therefore requires a rather strong prior parameterization and additional prior information.

S. A. Gritsenko: *What does Epov mean when he writes to me: "For about two years now we have moved away from two-dimensional formulations toward quasi-three-dimensional ones (accounting for the actual tool body resting against the borehole wall and determining not only transversely isotropic electrical resistivity, but also a second electrical parameter, relative permittivity and its frequency dispersion). More recently, we have also found an efficient solution to the inverse problem for spontaneous potential using sigmoid functions."*

GPT-5.6 Sol: *I interpret Epov's letter as briefly describing two different directions in a single paragraph; they should not be conflated.*

The first part concerns electromagnetic logging. Previously, their calculations were often 2D axisymmetric: the tool was assumed to be on the borehole axis and the geometry independent of azimuth. If the real tool lies against the borehole wall, this symmetry is broken. The field can no longer be described completely using only coordinates r and z; the position of the tool body in the borehole cross-section must also be taken into account. Hence his phrase "quasi-three-dimensional formulation." In a 2024 publication by Epov and coauthors, the computational

domain is indeed three-dimensional, the governing equation is a vector Helmholtz equation, the actual geometry of a three-coil tool is used, and electrical conductivity is represented by a tensor. (Epov et al., 2024)

Thus, the first part of the letter means approximately the following:

actual 3D tool–borehole geometry + anisotropic medium → electromagnetic signals → inversion for electrical properties.

Moreover, they recover more than a single resistivity. For a transversely isotropic medium, resistivity is effectively described by at least two quantities, for example the horizontal and vertical resistivities:

$$\rho_h(z), \rho_v(z).$$

In addition, they aim to determine

$$\varepsilon_r(\omega, z),$$

that is, relative permittivity and its frequency dispersion. The ability of electromagnetic signals to estimate conductivity and dielectric permittivity jointly had already been demonstrated by this school in work on 2D inversion of high-frequency electromagnetic logs. (Glinskikh et al., 2013)

Thus, the first part of the letter says, in effect, that they increased both the geometric realism of the forward problem and the number of physical parameters to be estimated.

The second phrase, however,

"More recently, we have also found an efficient solution to the inverse problem for spontaneous potential using sigmoid functions"

is, in my view, a separate SP problem rather than a continuation of the quasi-3D electromagnetic inversion.

There is an important subtlety here. In the published 2022 paper by Epov and Glinskikh, sigmoid functions are used primarily to represent spatial changes in resistivity and other coefficients smoothly in the SP forward problem. The authors explain explicitly that step functions are replaced by sigmoids because the governing equation contains derivatives; as the sigmoid steepness increases, the sigmoid approaches a step function. (Epov et al., 2022)

In the 2024 work by Glinskikh and Epov, an SP inverse algorithm is indeed reported, based on approximation of the measured signals by simple analytical functions followed by estimation of formation-water salinity. This is stated explicitly in the Russian Science Foundation project report. (RSF, 2026)

I would therefore interpret the entire letter as follows:

1. Electromagnetic logging: they moved from axisymmetric 2D models to models that account for the actual three-dimensional geometry of the tool and its eccentric position; in inversion they aim to determine tensor resistivity properties and $\varepsilon_r(\omega)$.

2. SP: separately, they developed a faster way to interpret the spontaneous-potential curve, using analytical—probably sigmoid—approximations of the signal or model parameters instead of a computationally expensive full inversion.

One more important qualification: the letter itself does not establish that sigmoid functions enter directly into the final inverse formula used in 2024–2026. In the open 2022 publication, sigmoids clearly refer to parameterization of the forward-problem coefficients, whereas the 2024 inverse-problem description refers more broadly to "simple analytical functions." It is therefore possible that Epov's personal letter briefly combines several recent developments of the group.

In other words, I would not read his statement as describing one single "quasi-3D inverse problem." It is more likely two consecutive achievements: a new generation of electromagnetic inversion and, separately, a new fast SP inversion.

S. A. Gritsenko: *What does this mean in terms of measurements—the observation functions?*

GPT-5.6 Sol: *In terms of observation functions, Epov's statement means that in the first problem they measure not a single depth curve but a set of one-dimensional curves, some of which additionally differ by frequency and by tool configuration.*

For VEMKZ, SKL-type tools measure phase differences and ratios of induced-EMF amplitudes in receiver coils. Joint inversion of these signals is used to determine the frequency dependences of electrical conductivity $\sigma(f)$ and relative dielectric permittivity $\varepsilon(f)$. The group's paper lists frequencies of 0.875, 1.5, 3.7, 7.0, and 14 MHz. (Epov et al., 2019)

Thus, the observations can be written approximately as

$$\Delta\Phi_i(z),$$

$$A_i(z),$$

where i denotes the frequency or sonde index.

With several frequencies, one effectively has a family of functions:

$$\Delta\Phi(z, f_i),$$

$$A(z, f_i).$$

At discrete frequencies these are several one-dimensional curves versus z. If frequency is treated conceptually as a continuous variable, they become two two-dimensional functions:

$$\Delta\Phi(z, f),$$

$$A(z, f).$$

It is precisely the frequency dependence that makes it possible to estimate the dispersion of

$$\sigma = \sigma(z, f),$$

$$\varepsilon = \varepsilon(z, f).$$

In terms of information dimensionality, this situation is therefore much closer to yours than the SP problem is.

In addition, to characterize anisotropy they use different types of logging tools with different sensitivities to components of the conductivity tensor. Their paper states explicitly that VEMKZ/VIKIZ is predominantly sensitive to the tangential component, BKZ to the vertical component, and BK to the radial component; the joint logging suite makes it possible to estimate elements of the diagonal conductivity tensor. (Epov et al., 2019)

The observations for this part can therefore be represented as a vector of functions:

$$D(z) = \{D_{BKZ}(z), D_{BK}(z), \Delta\Phi_1(z), A_1(z), \dots, \Delta\Phi_N(z), A_N(z)\}.$$

Thus, at each depth z several independent measured quantities are available.

In the newer quasi-3D formulation described by Epov, the tool body lying against the borehole wall probably introduces an additional dependence of the signals on the spatial orientation of the tool. The letter itself, however, does not specify exactly how many channels or which components are used; that requires a specific publication or instrument description.

For the second problem—SP—the data are much poorer:

$$SP(z)$$

is a single one-dimensional function of depth.

Thus, in terms of data, the two parts of his letter differ sharply:

quasi-3D electromagnetic problem: many channels × many frequencies × depth, effectively a large array of the form

$$D_k(z, f),$$

possibly with an additional orientation dependence;

SP inverse problem: a single curve

$$SP(z).$$

This is why the quasi-3D electromagnetic problem can potentially recover several parameters—for example ρ_h, ρ_v, and $\varepsilon(f)$—because the measurements are multichannel and multifrequency.

The SP problem based on a single curve depends much more strongly on prior constraints or a special low-dimensional parameterization.

To compare very briefly with your VSP problem:

in your case: $u(x,t)$ is one two-dimensional function;

in their VEMKZ data: the set of functions $\Delta\Phi_i(z)$ and $A_i(z)$, when frequency is taken into account, can be viewed as several functions of the two variables z and f;

in their SP problem: one function $SP(z)$.

So, in information dimensionality, the first part of Epov's letter is indeed much richer than we initially discussed.

S. A. Gritsenko: *How do the differential equations of the field differ in my problem and theirs?*

GPT-5.6 Sol: *Yes. If we compare the field equations themselves, there are two entirely different cases in Epov's work: electromagnetic logging and SP. The first is closer to your problem, but even there the differences are fundamental.*

Your problem uses a dynamic viscoelastic wave equation:

For the Maxwell model it is convenient to write it as the system

$$\rho u_{tt} = \frac{\partial \sigma_m}{\partial x},$$

$$\sigma_{m,t} + \alpha_R \sigma_m = M u_{xt}, \alpha_R = \frac{1}{\tau_R}.$$

Here $u(x,t)$ is the observed mechanical wavefield, $M(x)$ is the modulus, and $\tau_R(x)$ is the relaxation time.

For locally constant α_R, stress can be eliminated to give an equation of the form

$$\frac{\partial}{\partial x}(M u_{xt}) = \rho(u_{ttt} + \alpha_R u_{tt}).$$

Thus, your equation contains both x and t explicitly.

This is a nonstationary dynamic problem. The wave actually propagates in time, so initial conditions, propagation velocity, attenuation, and relaxation are essential.

Most importantly for the inverse problem, the unknowns $M(x)$ and $\alpha_R(x)$ occur as coefficients of the same equation, while the full field $u(x,t)$ is measured.

In Epov's electromagnetic-logging problem, the governing equations are Maxwell's equations:

$$\nabla \times E = \frac{-\partial B}{\partial t},$$

$$\nabla \times H = J_s + \sigma E + \frac{\partial D}{\partial t},$$

where

$$D = \varepsilon E, B = \mu H.$$

Here the field is vector-valued:

$$E = (E_x, E_y, E_z).$$

Under anisotropy, electrical conductivity is also no longer a scalar but a tensor

$$\sigma.$$

It is this tensor conductivity and the actual three-dimensional tool geometry that Epov's group incorporates into a vector finite-element calculation. (Epov et al., 2024)

Because the transmitter operates at a prescribed frequency, they usually pass to a harmonic representation

$$E(r,t) = \Re\{E(r,\omega)e^{i\omega t}\}.$$

The explicit time variable t then disappears, and Maxwell's system becomes a vector Helmholtz-type equation:

$$\nabla \times (\mu^{-1}\nabla \times E) - (\omega^2\varepsilon - i\omega\sigma)E = F.$$

The sign of the $i\omega\sigma$ term depends on the adopted convention $e^{\pm i\omega t}$, but this is not important here.

Thus, for each frequency one solves the spatial problem

$$E = E(x,y,z;\omega).$$

There is a very important difference from your case:

In your problem:

$$u = u(x,t)$$

and t is an actual independent coordinate of the observed field.

After transformation to the frequency domain, their field is

$$E = E(x,y,z;\omega),$$

where ω is a parameter of an individual experiment.

If many frequencies are used, they obtain the set

$$E(r,\omega_1), E(r,\omega_2), \ldots$$

and can regard it as spectral-spatial data. This is nevertheless not exactly the same as your directly recorded $u(x,t)$.

Moreover, they do not measure the complete field $E(x,y,z;\omega)$. They calculate it in the forward problem, while the instrument records induced EMF in several receiver coils at different depths. The 2024 paper states explicitly that the full vector field is calculated, whereas the observed quantities are induced EMFs in two receiver coils as the tool moves along the borehole. (Epov et al., 2024)

Thus,

In your case, the field is practically the data,

whereas in their case

the field is mainly an intermediate solution of the forward problem.

This is a very substantial difference.

Their SP problem is different again:

For spontaneous potential, the governing equation is not Maxwell's equations in wave form but the Poisson equation:

$$\nabla \cdot (\sigma \nabla \varphi) = \nabla \cdot J_s.$$

Or, in terms of resistivity:

$$\nabla \cdot \left(\frac{1}{\rho}\nabla\varphi\right) = \frac{k_B T}{e}\nabla \cdot$$

This is the equation used by their group for SP modeling. (Danilovskiy et al., 2026)

In the axisymmetric case:

$$\varphi = \varphi(r,z),$$

and time is absent altogether.

This is a stationary elliptic equation.

Thus, the three problems belong to different mathematical classes:

	Your VSP	Epov electromagnetic logging	Epov SP
Field	$u(x,t)$	$E(x,y,z;\omega)$	$\varphi(r,z)$
Field type	scalar	vector	scalar
Time	present explicitly	usually replaced by frequency ω	absent

	Your VSP	Epov electromagnetic logging	Epov SP
PDE type	wave equation with relaxation	Maxwell system / vector Helmholtz equation	Poisson equation
Forward-model dimensionality	1D space + time	3D space	2D (r, z)
Main unknown medium parameters	M, α_R	σ, ε	ρ, ρ_w, T_+
Is the full field observed?	practically yes: $u(x,t)$	no	no
Observation	$u(x,t)$	coil EMF along z, several frequencies	$SP(z)$

The most fundamental distinction, however, appears in the inverse problem:

In your case, the differential equation can be considered at every point of the recorded field:

$$L(M, \alpha_R)u = 0.$$

You know $u(x,t)$ and can therefore compute

$$u_x, u_{xx}, u_{tt}, u_{ttt}, \ldots$$

and seek the equation coefficients directly.

You use the PDE structure first to eliminate α_R and determine $M(x)$, and then to determine $\alpha_R(x)$.

Thus:

$$u(x,t) \rightarrow \text{coefficients of the PDE}$$

without necessarily performing repeated full-field forward simulations.

In Epov's electromagnetic problem, the scheme is

$$\{\sigma, \varepsilon\} \rightarrow E(x, y, z; \omega) \rightarrow \text{receiver EMF}.$$

But experimentally they do not know $E(x, y, z; \omega)$. Only several functionals of that field—the receiver-coil signals—are known.

Therefore, one cannot simply take spatial derivatives of the full measured E and obtain σ and ε directly from Maxwell's equations, as you do with u.

They must solve the inverse problem approximately according to the scheme

$$\{\sigma, \varepsilon\} \rightarrow \text{solve Maxwell} \rightarrow D_{calc} \rightarrow D_{obs} - D_{calc} \rightarrow \text{update model}.$$

For SP, the difference is even stronger:

There,

$$\nabla \cdot (\sigma \nabla \varphi) = f,$$

but the full

$$\varphi(r, z),$$

is not known; only its trace along the borehole line is observed:

$$\varphi_{obs}(z).$$

Therefore, they do not know

$$\frac{\partial \varphi}{\partial r}$$

throughout the domain, let alone

$$\frac{\partial^2 \varphi}{\partial r^2}.$$

Consequently, the two-dimensional differential equation itself cannot be used directly for local recovery of its coefficients from the data.

This, in my view, is the deepest dividing line between your formulation and theirs.

In your case:

Almost the entire function entering the PDE is measured:

$$u(x, t).$$

Therefore, both independent coordinates of the field are known, and the PDE can be converted into a local equation for the unknown coefficients.

In their case:

The PDE contains

$$\varphi(r, z)$$

or

$$E(x, y, z),$$

but only a small part of these functions is measured.

They therefore have to reconstruct the unobserved field by solving the forward problem.

This is precisely why your inverse approach is so different: you effectively use the measured solution of the differential equation itself, whereas in their case only functionals or a trace of that solution are observed.

S. A. Gritsenko: *And they cannot avoid prior information, because otherwise the solution becomes non-unique.*

GPT-5.6 Sol: *Yes. In the general case, that is essentially correct.*

The reason is that their observations are incomplete relative to the dimensionality of the field and the medium. For example, in SP:

$$\varphi = \varphi(r, z)$$

is a two-dimensional field, while only

$$\varphi_{\text{obs}}(z).$$

is measured. Thus, only the trace of the two-dimensional field along the borehole line is known.

If one attempts, without additional constraints, to recover for example

$$\rho(r, z),$$

then the same curve $\varphi(z)$ can be produced by different two-dimensional distributions $\rho(r, z)$, salinity, invasion-zone structures, and other parameters. In a linear or linearized view this corresponds to an unobservable null space; more generally, the problem admits indistinguishable models and is non-unique.

Schematically:

$$m_1(r, z) \neq m_2(r, z),$$

while at the same time

$$F[m_1](z) = F[m_2](z),$$

where F is the forward operator that maps the medium model to the observed curve.

They therefore have to introduce prior constraints such as the number of formations, boundary positions, cylindrical geometry of invasion zones, transverse isotropy, admissible resistivity ranges, known borehole diameter, mud properties, solution smoothness, sigmoid transition shapes, and joint interpretation of several logging methods.

After this, instead of an arbitrary function

$$m(r, z)$$

they effectively seek a finite parameter set

$$m = \{m_1, m_2, \dots, m_N\}.$$

The problem can then become practically identifiable.

There is an important subtlety here: prior information addresses not only non-uniqueness, but often also instability.

Even when the solution is formally unique, a small perturbation in the data

$$\delta d$$

may cause a large perturbation in the model

$$\delta m.$$

Regularization is therefore used:

$$\min_m [\| F(m) - d \|^2 + \lambda R(m)],$$

where $R(m)$ imposes smoothness, proximity to a prior model, minimum norm, and so forth.

This is where the difference from your problem becomes very substantial.

In your case, the full field is observed:

$$u(x,t),$$

while only two one-dimensional functions are recovered:

$$v(x), \tau_R(x).$$

Thus, the data are much richer relative to the unknowns. Moreover, you use the differential equation itself to separate the coefficients locally.

Therefore, a prior model such as “there must be 10 layers here” or “the boundaries must be sigmoid” is not an essential part of your formulation.

This can be stated very briefly:

In their problem, prior information supplies information missing from the measurements.

In your problem:

Much of the required information is contained in the two-dimensional function $u(x,t)$ itself.

This is perhaps one of the main mathematical differences between the two approaches.

S. A. Gritsenko: *You listed too many prior conditions. What is needed is a single mathematical condition.*

GPT-5.6 Sol: *Yes. Everything I listed earlier consists of particular ways of implementing one general mathematical condition.*

It can be written as

$$m(r,z) \in M_{adm},$$

where M_{adm} is a prescribed admissible class of two-dimensional models on which the observation operator F must be one-to-one:

$$F[m_1] = F[m_2] \Rightarrow m_1 = m_2.$$

Thus, the single essential prior condition is:

the unknown two-dimensional medium must belong to a restricted model class for which the one-dimensional observations determine the model uniquely.

In the context of our discussion, this class can be written even more transparently as

$$m(r,z) = G\big(r, q(z)\big),$$

where the dependence on r is prescribed a priori by the function G and only one unknown function remains:

$$q = q(z).$$

Then the original problem

$$d(z) = F[m(r,z)]$$

becomes

$$d(z) = \tilde{F}[q(z)].$$

and the uniqueness condition becomes simply

$$\tilde{F}[q_1] = \tilde{F}[q_2] \Rightarrow q_1 = q_2.$$

In my view, this is the single mathematical formulation you were looking for: an a priori reduction of the class of unknown two-dimensional functions to a class parameterized uniquely by a smaller number of functions or parameters.

Your problem does not require this kind of artificial dimensional reduction of the medium: $u(x,t)$ already contains the additional independent variable t, while the unknowns $v(x)$ and $\tau_R(x)$ are one-dimensional functions.

S. A. Gritsenko: *Or should one instead find functions of the medium that are determined by their observations? A considerable body of work, including the classical studies you listed, is devoted to exactly this.*

GPT-5.6 Sol: *Yes. That is a more precise formulation.*

It is not necessary to attempt to recover the entire two-dimensional medium $m(r,z)$. One can seek a function of the medium

$$q(z) = G[m(r,z)],$$

that is uniquely, or at least sufficiently stably, determined by the observed curve

$$d(z).$$

Thus, instead of the problem

$$d(z) \to m(r,z),$$

which is usually non-unique, one formulates the problem

$$d(z) \to q(z),$$

where $q(z)$ is not the entire medium but an observable characteristic of it.

A substantial part of both classical and modern electrical geophysics is devoted to exactly this idea.

For example, in the SP method Revil and coauthors recover from the measured potential not the full distribution of electrical properties, but the shape and depth of the piezometric surface $h(x)$. Thus, one observation function is converted into one physically meaningful function of the medium. (Revil et al., 2003)

Another example is Minsley, Sogade, and Morgan. They abandon direct recovery of the complete geoelectric model and instead seek the distribution of SP sources, that is, the function directly responsible for the observed potential. The authors explicitly note that the full problem is non-unique, so they recover a particular class of sources compatible with the observations. (Minsley et al., 2007)

In classical electrical geophysics, this idea is even older. In vertical electrical sounding, combinations of layer parameters are often much better determined than thickness h and resistivity ρ separately. For example, the Dar Zarrouk parameters are

$$S = \frac{h}{\rho}$$

— longitudinal conductance,

and

$$T = h\rho$$

— transverse resistance.

For equivalent layered models, the individual h and ρ may be unstable or ambiguous, whereas S or T can be determined much more reliably. This is a

classical example of an inverse problem that may not determine the entire medium but can determine a functional of it quite well.

Mathematically, it is therefore more accurate to formulate the problem as follows:

$$d = F[m],$$

where m is the complete medium model.

Even if F^{-1} does not exist uniquely, another operator G may exist such that

$$q = G[m]$$

has the same value for all models producing the same observations:

$$F[m_1] = F[m_2] \Rightarrow G[m_1] = G[m_2].$$

Then q is an identifiable characteristic of the medium.

This, in my view, is the key point.

For Epov's problem, the right question may therefore be not:

"How can the two-dimensional medium be recovered from $SP(z)$?"

but:

"Which one-dimensional function $q(z)$ related to the two-dimensional medium is uniquely determined by the curve $SP(z)$?"

Philosophically, this is already very close to your problem. You too have effectively identified not the entire microstructure of the viscoelastic medium, but macroscopic functions such as

$$v(x), \tau_R(x),$$

that are determined from the observed field.

The difference is that in your case they are derived directly from the PDE and the full field $u(x,t)$, whereas in electrical geophysics such identifiable functions are often sought through properties of the integral operator, model equivalence, and physically meaningful combinations of parameters.

In other words, you have just formulated a more general principle:

If the medium as a whole is not identifiable, one should seek identifiable functions of the medium.

This is indeed one of the central ideas of inverse-problem theory.

S. A. Gritsenko: *Then why does Epov's group not use this general principle?*

GPT-5.6 Sol: *Judging from the published work, I would now say that Epov's group does use this general principle, although they do not formulate it in the abstract form at which we have arrived.*

Mathematically, we formulated the principle as follows. Suppose the complete medium m is not uniquely determined by the data d:

$$d = F[m],$$

and there exist different media

$$m_1 \neq m_2, F[m_1] = F[m_2].$$

Then, instead of recovering the entire m, one should seek a characteristic

$$q = G[m],$$

for which

$$F[m_1] = F[m_2] \Rightarrow G[m_1] = G[m_2].$$

In other words, seek what is actually identifiable from the data.

Several variants of this idea can in fact be seen in work by Epov's group.

In the 2024 SP work by Glinskikh and Epov, they effectively do exactly this: from a single curve $SP(z)$ they do not attempt to recover the full two-dimensional structure $\rho(r,z), \rho_w(r,z), T_{+(r,z)}$, but instead use an analytical approximation of the log to rapidly estimate formation-water salinity. In practice:

$$SP(z) \rightarrow C_w(z) \text{or parameters } C_w,$$

rather than

$$SP(z) \rightarrow \text{the full two-dimensional medium.}$$

The title of the paper itself makes this explicit: "Approximation of … logs for rapid estimation of formation-water salinity." (RSF, 2026)

Likewise, in the 2026 paper they do not claim recovery of an arbitrary two-dimensional medium from SP. They use a physical 2D forward model, but from field data they primarily estimate formation-water salinity and the cation transference number. (Danilovskiy et al., 2026)

The earlier line of work shows similar thinking. For example, in 2005 Glinskikh and Epov studied spatial sensitivity of signals and introduced pseudogeometric factors, thereby examining which integral characteristics of the medium are actually reflected in the measured signal. (Glinskikh, Epov, 2005)

In 2009 they replaced a detailed layered model by a continuous conductivity function and noted the role of the mean conductivity. (Glinskikh, Epov, 2009)

The question should therefore be posed somewhat differently:

why do they continue to recover fairly detailed 2D and quasi-3D models alongside the search for identifiable characteristics?

The answer is already visible in their work. Their electromagnetic data are much richer than a single SP curve: relative amplitudes, phase shifts, multiple sondes, and multiple frequencies are used simultaneously. Joint inversion of amplitudes and phases allowed them to recover the spatial conductivity distribution and estimate dielectric permittivity already in their 2013 work. (Glinskikh et al., 2013)

In later work they jointly use different electrical and electromagnetic methods to estimate anisotropy, dielectric permittivity, and other parameters. (Epov et al., 2019)

Thus, with a rich observation set they follow the path

$$D_1(z), D_2(z), \dots, D_N(z) \rightarrow \text{parameterized 2D/3D model.}$$

whereas with a poor observation such as a single $SP(z)$ they effectively move toward

$$SP(z) \rightarrow \text{one or more identifiable characteristics.}$$

So there is no contradiction.

There is, however, a subtler point. I did not find in their work an explicit general formulation of the type "find all functionals $G[m]$ that are invariant over the class of equivalent models $F[m] = d$." Their publications usually begin from a concrete physical objective: resistivity, dielectric permittivity, salinity, transference number, invasion zones, and so forth. They select the physically required parameters and construct an inversion for them.

We, however, have now formulated the problem at a more general level:

Do not first choose the desired parameter; first determine what the data contain uniquely at all.

That is a substantially different order of reasoning.

This is precisely where, in my view, an interesting point for discussion with Epov arises. One could ask him directly:

"Have you considered not the problem of recovering a predefined parameterized two-dimensional model, but the problem of finding those functionals of the two-dimensional medium that are uniquely determined directly by the measured one-dimensional or multichannel well-log data?"

This is already a very specific mathematical question, and Epov's answer could show how far their school has gone specifically in the theory of identifiable characteristics rather than only in numerical inversion.

S. A. Gritsenko: *I think they did not use this approach because it is mathematically more difficult, but it does allow the uniqueness issue to be addressed.*

GPT-5.6 Sol: *Yes, that is a plausible explanation—but I would state it more cautiously: this is our inference, not an established fact about the motivations of Epov's group.*

Mathematically, it is indeed simpler to start from a preselected parameterization of the medium:

$$m(r, z; p)$$

and seek a finite parameter vector p by minimizing

$$\| F[m(p)] - d \|^2.$$

Here, the uniqueness issue is partly "resolved" by the choice of a restricted model family: a huge number of possible $m(r, z)$ are excluded in advance.

A deeper formulation is to ask instead:

$$d = F[m],$$

and, without assuming a particular form of m in advance, study the equivalence classes

$$[m]_F = \{\tilde{m} : F[\tilde{m}] = F[m]\}.$$

Then seek functions or functionals $G[m]$ that are constant over the entire class:

$$F[m_1] = F[m_2] \Rightarrow G[m_1] = G[m_2].$$

Then the quantity

$$q = G[m]$$

is uniquely determined by the data even though m itself is not.

This is mathematically much more difficult, because one must study not merely the performance of a numerical algorithm but the structure of the operator F itself: its null space, equivalent models, sensitivity, and invariants.

The resulting statement, however, is much stronger. One can say rigorously:

these observations do not determine the complete two-dimensional medium, but this particular function of the medium is uniquely determined.

This is the value of classical results such as the Dar Zarrouk parameters: individual ρ_i and h_i may be equivalent, whereas certain combinations of them are much better constrained by the observations.

For Epov's problem, this might look as follows:

$$SP(z) = F[m(r, z)]$$

and instead of requiring recovery of $m(r, z)$, formulate the problem:

$$\text{find } q(z) = G[m(r, z)]$$

such that

$$F[m_1] = F[m_2] \Rightarrow q_1(z) = q_2(z).$$

This directly addresses the uniqueness question.

I would actually formulate it slightly more precisely than you did: such an approach does not necessarily make the full inverse problem unique. It allows the uniquely identifiable part of the solution to be isolated. The non-identifiable part of the medium remains non-unique, but we stop asking the data to provide information that they do not physically contain.

This is very close to what your method achieves: you do not attempt to reconstruct the microscopic structure of the viscoelastic medium, but recover the macroscopic parameters $v(x)$ and $\tau_R(x)$ that are contained in the measured dynamic field.

S. A. Gritsenko: *Can the same kind of locality as in my method be constructed at all in Epov's problem, so that a finite segment of the input data determines the medium parameter at the corresponding point?*

GPT-5.6 Sol: For the original SP problem, in the strict sense, no. This is an important clarification: the absence of such locality in Epov's problem is not simply a deficiency of the chosen algorithm, but follows from the physical-mathematical nature of the problem.

The SP field is described by an elliptic equation of the form

$$\nabla \cdot (\sigma \nabla \varphi) = \nabla \cdot J_s$$

If the medium parameters are changed locally in a small region, the potential perturbation can be represented through a Green's function:

$$\delta\varphi(x) = \int G(x, y)\, q(y)\, dy$$

Even if $q(y)$ is nonzero only in a small region, the Green's function of an elliptic operator generally does not become exactly zero beyond some finite distance. A local change in the medium therefore affects, in general, the potential at all observation points. The influence may decrease rapidly with distance and become practically small, but there is no strict finite data window of the form

$$[z - L, z + L] \rightarrow m(z)$$

for this problem.

Thus, locality of your type is physically impossible in the original Epov problem. It would therefore be incorrect to regard the absence of such a local operator as a deficiency of Epov's method.

The sigmoid approximation creates a certain effective localization: an individual formation predominantly changes the shape of the log in the

corresponding depth interval. But this is not strict locality of the original elliptic PDE. Sigmoid functions have extended tails, and the parameters of all contributions are determined jointly over the considered log.

This is precisely where an important similarity between Epov's method and FWI appears.

In FWI, an individual local change in the medium also has a certain spatial sensitivity pattern in the data. But the inverse model is not recovered locally; it is sought from a global misfit functional:

$$\Phi(m) = \sum |d_{obs} - d_{calc}(m)|^2$$

Thus, local physical sensitivity of the forward problem does not automatically become a local inverse operator.

The Glinskikh–Epov logic is analogous:

the entire SP log under consideration → global misfit → joint fitting of the sigmoid parameters.

The computational implementation is substantially different: FWI uses the original PDE to calculate the response, whereas Glinskikh–Epov replace repeated forward calculations by a parametric analytical approximation of the SP log. Their method therefore should not be called FWI in the strict sense.

But the inversion structures are related:

FWI — global inversion through misfit minimization; Glinskikh–Epov — global parametric inversion based on an analytical approximation of the log; Gritsenko — local identification of coefficients directly from the observed field.

In your method, the finite observation base is set by the procedure used to calculate derivatives, and the parameter at a given point is determined from that base. Locality is therefore a property of the inverse algorithm itself.

Identifiability First, Algorithm Second: Comparison with FWI

S. A. Gritsenko: *This is more important and more correct: first determine what in the medium is constrained by the input data, and then recover exactly that. Then genuine inversion algorithms arise, instead of attempts to recover the entire medium without understanding what can actually be determined. Otherwise one gets methods such as FWI with outrageously expensive brute-force computations that are not justified by the information content of the data.*

GPT-5.6 Sol: *Yes. I would only qualify your formulation slightly: the problem with FWI is not that the method is "unjustified" in itself. FWI has a rigorous formulation as PDE-constrained optimization. But optimization by itself does not answer the more*

fundamental question of which characteristics of the medium are uniquely contained in the data. Modern reviews explicitly list non-uniqueness, nonlinearity, local minima, and cycle skipping among the central challenges of FWI. (Métivier, Brossier, 2026)

In my view, you are formulating a more appropriate sequence for solving an inverse problem.

Let the observations be

$$d = F[m],$$

where m is the complete model of the medium.

First, one should establish model equivalence:

$$m_1 \sim m_2 \Leftrightarrow F[m_1] = F[m_2].$$

That is, determine which different media are fundamentally indistinguishable in the given experiment.

Then, one need not necessarily seek m itself, but rather functions or functionals

$$q = G[m],$$

that are identical for all equivalent models:

$$F[m_1] = F[m_2] \Rightarrow G[m_1] = G[m_2].$$

Then q is precisely the quantity physically contained in the input data.

Only after that does it make sense to construct an algorithm

$$d \to q.$$

I would call this an "identifiability before inversion" approach.

It is stronger conceptually than the usual scheme:

$$\text{choose model } m \to \min_m \| F[m] - d \|^2.$$

because in the latter case the optimizer may find some model m that reproduces the data extremely well,

$$F[m\ ] \approx d,$$

but this does not imply that this particular model is determined by the data. There may be an entire family

$$m_1, m_2, \dots, m_N$$

with practically identical synthetic data.

Then regularization

$$\min_m \{\| F[m] - d \|^2 + \lambda R[m]\}$$

effectively selects one representative from the family of admissible solutions. The selected model depends not only on the observations but also on $R[m]$, the starting model, the parameterization, and other assumptions. This is a normal and deliberate part of modern inversion, but it should not be confused with proof that every recovered detail is determined by the data themselves. In inverse-problem theory, parameterization, resolution, and uncertainty are themselves fundamental and distinct issues. (Tarantola, 2005)

This makes your criticism of the computational side of FWI particularly understandable. A typical FWI repeatedly performs

$$m^{(k)} \rightarrow \text{wave-equation solution} \rightarrow d_{calc}^{(k)} \rightarrow d_{calc}^{(k)} - d_{obs} \rightarrow \nabla J \rightarrow m^{(k+1)}.$$

That is, the forward and adjoint wave equations are solved repeatedly for many sources and iterations. The high dimensionality of the model space and repeated full forward solutions are precisely what make FWI computationally expensive. (Malinowski et al., 2011)

The problem remains that if a target parameter is poorly identifiable from the data, increasing the number of iterations does not add new information. It only searches more thoroughly within the already chosen model formulation.

I would therefore separate two entirely different questions:

What can be determined from the data?

and only then

How can this be determined most efficiently?

In FWI, these two questions are often combined in practice: one chooses $v(x,z)$, V_P, V_S, ρ, Q, or another parameter set in advance and then attempts to recover it by optimization. In multiparameter FWI, the problem is especially pronounced because of parameter cross-talk and trade-offs. Modern literature therefore devotes considerable attention to parameterization, starting models, regularization, and methods for mitigating non-uniqueness and nonlinearity. (Métivier, Brossier, 2026)

Your approach proceeds conceptually in the opposite order. You began with the equation and the measured $u(x,t)$, established which coefficients could be separated from it, obtained first $M(x)$ and then $\alpha_R(x)$, and only then did the concrete algorithm emerge. In this sense, the algorithm follows from an analysis of what is identifiable rather than serving as a way to guess the medium by searching over models.

I would formulate your idea almost as a general methodological principle:

An inverse problem should begin not with the choice of an algorithm for reconstructing a model, but with determining which functions of the medium are uniquely and stably determined by the available observations. Inversion algorithms should then be constructed specifically for those identifiable functions.

This already goes beyond comparing your work with Epov's. It is a self-contained methodological position on how geophysical inverse problems should be formulated.

S. A. Gritsenko: *But for now, FWI is winning in the literature.*

GPT-5.6 Sol: *Yes. As of 2026, FWI dominates publications, methodological development, and industrial attention, especially in seismics. This is evident even from the very recent review by Métivier & Brossier: they examine nearly forty years of FWI development and note that some variants have already become an industrial standard.* (Métivier, Brossier, 2026)

But this does not mean that FWI has "solved" the uniqueness problem. On the contrary, the same review explicitly identifies ill-posedness, non-convexity, nonlinearity, and solution non-uniqueness as major challenges. Over the decades, a vast body of work has therefore focused on making optimization work despite these fundamental limitations. (Métivier, Brossier, 2026)

A very recent 2026 study of FWI uncertainty makes the point even more strongly: real tomographic problems remain highly non-unique because of limited acquisition geometry, noise, nonlinearity, and model underdetermination. The authors therefore move toward estimating a family of possible models rather than treating the optimal model found by inversion as the one true medium. (Zhao, Curtis, 2026)

This produces an interesting historical situation.

The main FWI line is:

$$m \to F[m] \to \| F[m] - d \|^2 \to m \quad .$$

That is, one first decides, "we will recover the model m," and then spends forty years refining ways of finding it in an extremely high-dimensional model space.

The principle we have arrived at begins one step earlier:

$$d = F[m]$$

and first asks:

$$\text{What exactly in } m \text{ is determined by these } d?$$

Only after answering that question do we introduce

$$q = G[m]$$

and construct

$$d \to q.$$

This is genuinely a different order of solving the problem.

Modern literature is gradually moving toward this issue, although it largely remains within the FWI paradigm. For example, a 2025 paper is literally titled “Designing full waveform inverse problems”: it optimizes the acquisition system and the model parameterization jointly in order to reduce the dimensionality of the inverse problem and increase the information content of the data. The authors explicitly note that standard discretization creates a high-dimensional, effectively over-parameterized model space. (Mercier et al., 2025)

This is already a step toward your line of reasoning, but it is still not a full transition to the question

$$F[m_1] = F[m_2] \Rightarrow \text{what must be identical in } m_1, m_2?$$

Why does FWI still “win”? I think the reasons are mainly practical. It produces an attractive spatial model, naturally uses existing numerical wave-equation solvers, scales well on powerful GPUs, fits into industrial processing workflows, and allows the result to be improved progressively. An enormous technological ecosystem has therefore grown around it.

Searching for identifiable functions of the medium, by contrast, requires prior mathematical analysis of the specific operator F. One cannot simply take a universal optimizer and apply it to every problem. For each data type, one must determine the equivalence structure, the operator’s null space, invariants, and stably identifiable parameter combinations. This is much harder theoretically.

But the gain is fundamental:

We know in advance that what we recover is precisely what is contained in the data.

I would therefore distinguish two things: today FWI wins technologically and in publication volume; whether it is the most fundamental way to formulate an inverse problem remains an open question.

It is also interesting that modern FWI literature increasingly has to return to questions of parameterization, resolution, information content, and uncertainty—issues that are all parts of the more general question “what do the data actually determine?” (Zhao, Curtis, 2026)

S. A. Gritsenko: *Recently a third path has appeared, associated with neural networks and their training. What are its prospects?*

GPT-5.6 Sol: *The neural-network path has major prospects, but I would distinguish computational success of a neural network from solution of the inverse problem itself. In the context of our discussion, this distinction is fundamental.*

Let the forward problem be

$$d = F[m],$$

where m is the medium and d denotes the observations. In conventional supervised training, thousands of pairs are generated:

$$m_i \to d_i = F[m_i],$$

and then a network N_θ is trained to approximate the inverse mapping

$$N_\theta(d) \approx m.$$

After training, this is extremely attractive:

$$d_{obs} \overset{N_\theta}{\to} m$$

can be evaluated almost instantaneously. This post-training speed is regarded as one of the main advantages of AI inversion; recent reviews point to the promise of such methods for repeated and near-real-time tomography. (Ahmed, Güneyli, 2025)

But the same uniqueness issue we have been discussing immediately arises.

Suppose that

$$m_1 \neq m_2,$$

but

$$F[m_1] = F[m_2] = d.$$

Then for a single input d an ordinary deterministic neural network must return one result:

$$N_\theta(d) = ?$$

It cannot physically return both m_1 and m_2 simultaneously.

Therefore, training has not removed the non-uniqueness of the original problem.

It has only taught the network to choose some representative from the set of equivalent models, according to which models dominated the training distribution.

This is extremely important. Prior information has not disappeared; it has simply moved from explicitly imposed constraints

$$R[m]$$

into the distribution of the training models.

This is why a neural network can perform exceptionally well on synthetic tests drawn from the same family of models used in training and then degrade markedly

when applied to different geology. Generalization and out-of-distribution behavior are now regarded as major limitations of geophysical AI. (Ahmed, Güneyli, 2025)

Moreover, if training uses mean-squared error, then under non-uniqueness the network tends, in a statistical sense, to approximate a conditional mean such as

$$N_\theta(d) \sim E[m \vee d],$$

and such an "average model," when the solution set is multimodal, may correspond to no real geological model at all.

I therefore see four different prospects for neural-network inversion.

1. Pure direct neural-network inversion $d \to m$. It is extremely fast after training and will be widely used where the model class is well constrained and a large representative training set is available. But as a fundamental solution of a general inverse problem it is the most vulnerable: the result depends on the training set, and non-uniqueness is hidden rather than eliminated.

2. Physics-informed approaches. A condition enforcing the differential equation is added to the loss:

$$J(\theta) = J_{data} + \lambda J_{PDE}.$$

This is much more reasonable: the network is no longer free to output an arbitrary model. Physics-informed seismic inversion is therefore developing rapidly. A 2025 review describes several ways of embedding physics—through the network architecture, data constraints, and the loss function—with the aim of reducing ambiguity and improving physical interpretability. (Song et al., 2025)

Even satisfaction of the PDE, however, does not prove uniqueness of the coefficients: two different media can satisfy the same PDE and produce the same accessible observations.

3. Generative and probabilistic networks. Here an important conceptual shift has occurred. Instead of

$$d \to m$$

one attempts to estimate

$$p(m \vee d),$$

that is, a set of possible media with their probabilities. This is more honest when the problem is non-unique. A recent 2026 review of Bayesian geophysical inversion specifically considers deep learning as a means of defining geological priors, accelerating the forward problem, and constructing probabilistic inversion. (Liu et al., 2026)

4. Neural operators and surrogate models. A network can be trained not on the inverse problem itself but on the forward operator F. Then, instead of thousands of expensive PDE solutions, one uses a fast neural surrogate:

$$m \overset{N_\theta}{\to} d.$$

It can then accelerate FWI, Bayesian inversion, or Monte Carlo methods. This path will probably be very important computationally, but again it accelerates the search; by itself it does not answer what is determined by the data. Recent reviews identify hybrid physics–ML methods as one of the main directions of further development. (Hosseinzadeh et al., 2025)

Now we come to the most interesting point in the context of our discussion.

I think there is a substantially stronger way to combine neural networks with the principle at which we have arrived.

First, one studies mathematically

$$d = F[m]$$

and finds a function

$$q = G[m],$$

for which it is proved, or at least established, that

$$F[m_1] = F[m_2] \Rightarrow G[m_1] = G[m_2].$$

That is, q is genuinely determined by the input data.

Only after that is the network trained:

$$N_\theta : d \to q.$$

The neural network then no longer tries to "guess the whole Earth." It performs a completely different role: it becomes a fast and stable evaluator of a mathematically identifiable characteristic of the medium.

The sequence is then:

Data → identifiability analysis → q → training of a fast inverse operator.

rather than

Data → huge network → full medium model.

In my view, this option has the greatest scientific promise.

The three directions we have discussed can then be arranged very clearly:

$$\text{FWI:} d \to \min_m \| F[m] - d \|^2$$

— one complete model is sought by optimization;

$$\text{pure AI: } d \to N_\theta(d) \approx m$$

— the network learns to output a model from examples;

$$\text{identifiability} + \text{AI: } d \to q, q = G[m]$$

— *first determine what is contained in the data, and then train the network to extract exactly that as quickly and stably as possible.*

The last path combines the strength of mathematics—uniqueness and physical meaning—with the strength of neural networks—a fast nonlinear mapping of very large data sets after training.

I therefore do not think neural networks will simply "defeat FWI" in the end. More likely, the boundaries will blur: physical PDEs, identifiability analysis, probabilistic uncertainty assessment, and learned operators will be combined. This movement toward hybrid, physics-consistent, uncertainty-aware systems is already visible in the 2025–2026 literature. (Hosseinzadeh et al., 2025)

A pure neural network $d \to m$ used without first answering the question "is m actually determined by these data?" has, in essence, the same logical weakness that you identified in FWI: it starts with an algorithm before the identifiability of the target quantity has been established.

3. GPT-5.6 Sol Conclusion

As the discussion continued, the main emphasis shifted from comparing specific algorithms to a more general question: what exactly about the medium is contained in the observations and can therefore, in principle, be recovered?

The first clarification concerns the dimensionality and nature of the data. For SP, the full forward problem is two-dimensional,

$$\varphi = \varphi(r, z),$$

but only one one-dimensional function is observed directly,

$$SP(z).$$

The vector electric field $E = -\nabla\varphi$ exists physically, but standard SP logging does not record its two components independently. Electromagnetic logging is richer: a family of amplitude and phase curves is measured for several sondes and frequencies, so the data can be represented as a set of functions $D_k(z, f)$; with azimuthal sensitivity, an additional orientation variable may be present. Thus, the two parts of Epov's letter refer to problems with substantially different information content: multichannel quasi-3D electromagnetic inversion and, separately, inversion of a single SP curve.

The second clarification concerns the differential equations themselves. In your problem, the observed field $u(x, t)$ is a solution of a nonstationary viscoelastic wave equation, and practically the whole field entering the PDE is available in the measurements. This makes it possible to use its derivatives directly to determine the coefficients $M(x)$ and $\alpha_R(x)$. In Epov's electromagnetic problem, the full vector field $E(x, y, z; \omega)$ is

an internal solution of Maxwell's equations and is not observed directly; only functionals of it—the receiver-coil EMFs—are recorded. In SP, only the trace of the two-dimensional potential field along the borehole line is observed. The unobserved part of the field therefore has to be reconstructed by solving the forward problem.

This leads to the central conclusion concerning prior information and uniqueness. If

$$d = F[m]$$

and there exist different models $m_1 \neq m_2$ for which

$$F[m_1] = F[m_2],$$

then the complete medium is not identifiable from the data. One approach is to restrict the admissible model class a priori so that the operator F becomes one-to-one on that class. A more fundamental approach is to seek a characteristic

$$q = G[m],$$

that is identical for all equivalent models:

$$F[m_1] = F[m_2] \Rightarrow G[m_1] = G[m_2].$$

Then q is an identifiable characteristic of the medium. In this sense, classical Dar Zarrouk parameters, recovery of the piezometric surface from SP, inversion of source distributions, and similar results are fundamentally important: they show that even when the full model is non-unique, individual functions or functionals of the medium may be determined much better or even uniquely.

The principle formulated in the dialogue can therefore be written as follows:

First determine what is contained in the data; then construct an algorithm to recover exactly that.

Or, more formally:

$$d = F[m] \Rightarrow \text{model-equivalence analysis} \Rightarrow q = G[m] \Rightarrow d \to q.$$

This is the principle of "identifiability before inversion."

A detailed analysis of the Glinskikh–Epov 2024 paper made it possible to clarify the nature of their rapid SP inversion. The observed log is approximated by a superposition of analytical sigmoid functions whose parameters are determined jointly by least squares; the amplitude characteristics are then converted into formation-water salinity. This scheme is therefore primarily a finite-dimensional parametric inversion of the observed response using an analytical parameterization of the SP log, rather than direct local recovery of the coefficients of the original elliptic PDE.

The comparison revealed two additional fundamental differences from the viscoelastic VSP problem. The first concerns separation of the unknowns. In the VSP problem, both target quantities are medium parameters, and the first is determined independently of the second directly because of the structure of the original equation; the second parameter is determined afterward. In the SP problem considered here, there is no analogous separation of two simultaneously unknown physical medium parameters. The existing separation between signal shape and amplitude is not equivalent to this property, because the shape

parameters and amplitude are intermediate characteristics of the field representation, while the physical medium parameters are partly coupled and some of them are fixed from external information.

The second difference concerns locality. In the VSP problem, locality is a property of the inverse operator itself: a finite spatial base of the observed wavefield determines the medium parameter at the given point. For the original elliptic SP problem, such strict locality is impossible: a local change in the medium generally changes the potential throughout the observation domain, although its influence decreases with distance. Therefore, the absence of a local inversion in the Glinskikh–Epov method should not be regarded simply as a deficiency of the algorithm; it is related to the nature of the original elliptic field.

At the same time, it is precisely the global character of the inversion that relates their method to FWI. In both cases, the spatial sensitivity structure of the forward problem is not used in the form of a finite local recovery operator; the parameters are determined by joint minimization of an overall misfit over the data set. The difference is that FWI repeatedly solves the original wave equation during optimization, whereas Glinskikh–Epov use a much less expensive analytical parameterization of the SP log. It is therefore more accurate to speak not of FWI in the strict sense, but of an FWI-like logic of global inversion.

Against this background, FWI and neural-network methods occupy a different place. FWI is mathematically well founded as a PDE-constrained optimization problem, but minimization of the misfit

$$\min_{m} \| F[m] - d \|^2$$

does not by itself prove that the recovered model details are determined by the data. Regularization, parameterization, and the starting model select a specific representative from the set of models consistent with the observations. Modern FWI literature increasingly recognizes this issue through studies of resolution, parameterization, uncertainty, and families of admissible models.

A neural network provides a third path. After training, it can evaluate the mapping $d \to m$ extremely rapidly, but if the original problem is non-unique, training does not remove that non-uniqueness: prior information is effectively transferred into the training set. A deterministic network selects one characteristic answer, whereas probabilistic and generative methods attempt to describe the distribution $p(m \vee d)$.

The strongest approach appears to be a combination of these directions:

Identifiability + physical model + learned fast inverse operator.

That is, first determine mathematically the function $q = G[m]$ that is genuinely contained in the data, and then use an analytical algorithm, FWI, a neural network, or another computational method only as a means of recovering it efficiently. In this formulation, the computational algorithm no longer substitutes for the uniqueness question; it becomes the second stage, following analysis of the information content of the experiment.

Thus, comparison of the three approaches reveals not one but several independent methodological distinctions. One should first establish which characteristics of the medium are actually determined by the observations; then determine whether the physical structure of the problem permits separation of the unknown parameters; and finally determine whether the inverse mapping is physically local or necessarily global. Only after that should a specific computational algorithm be chosen—local formulas, global optimization, FWI, an analytical parameterization of the SP log, or a neural network. In the viscoelastic VSP problem, completeness of the observed field, sequential independence of the two target coefficients, and

locality of the inverse operator are interconnected properties of a single formulation. In the Glinskikh–Epov SP problem, the physics of the elliptic field excludes analogous strict locality, while coupled medium parameters and incomplete observations naturally lead to global parametric inversion.

4. Review by GPT-6.1 Sol of the Transcript of the Dialogue between S. A. Gritsenko and GPT-5.6 Sol

The transcript addresses an important problem of geophysical interpretation: which characteristics of the medium are determined by the input observations, and how should the target of an inverse problem be chosen? The discussion develops from specific electrical-logging methods to a general principle for formulating inverse problems, and then to a comparison of local seismic inversion, FWI, and neural-network methods.

The main substantive result of the dialogue is expressed by the following sequence: first establish what is determined by the observations, and only then construct an algorithm to recover exactly that. This order connects the physical experiment, the choice of target characteristics, and the computational method. It allows the information content of the result to be considered before the quality of fit between calculated and measured data is assessed.

Especially important is the idea of recovering functions or functionals of the medium rather than attempting to determine the entire medium. If different models are consistent with the same observations, some characteristics of these models may coincide or be determined much more reliably than others. Non-uniqueness of the full model therefore does not make the experiment useless; it calls for a more meaningful choice of what should be recovered. The discussion of model equivalence and classical parameter combinations in electrical geophysics helps clarify this idea.

The comparison with the work of M. I. Epov's group gives the discussion concrete content. It is important to distinguish the SP log, multichannel electromagnetic observations, and the internal seismic wavefield. These data provide different access to the spatial structure of the field and to medium properties. Parameterization of formations and invasion zones, analytical approximation of signals, and use of additional logging data are ways of organizing a particular inverse problem. Their methodological assessment should take into account which characteristics are extracted from the observations and which are imposed by the adopted model.

At the same time, Epov's letter does not justify the conclusion that his group does not study parameter identifiability or does not use related ideas. The reported industrial results deserve consideration, while the confidentiality of part of the work limits independent assessment. The approaches should be compared using available formulations and results, while preserving the distinction between developers' statements and published information.

S. A. Gritsenko's work provides, within this dialogue, a concrete example of constructing an inversion algorithm from the structure of the governing equation and an observed internal field. A two-stage procedure for recovering elastic and relaxation parameters is proposed, and numerical experiments are presented in which the prescribed characteristics of the viscoelastic medium are recovered. Derivation of the algorithm and numerical confirmation of its performance on the investigated models constitute an independent scientific result. For the subject of the transcript, the essential point is how the chosen characteristics of the medium are linked to the field and become targets of sequential recovery. A detailed mathematical verification of individual steps in the author's derivation belongs to a separate study.

The discussion of FWI raises an appropriate question about the relationship between computational solution and the information content of the data. Successful minimization of a misfit does not by itself show which details of the model are determined by the observations. At the same time, the description of FWI as "unjustified brute-force search" should be qualified: the method has a mathematical formulation and uses directed model updates. This clarification improves the comparison because it preserves the main issue—the justification of the inversion target—while allowing computational cost to be discussed accurately.

Neural-network methods continue the same methodological line. Training can accelerate the production of a result and transfer information from the training set into the algorithm. However, a fast network output does not by itself establish which part of that output is constrained by the specific observations. The principle of selecting identifiable characteristics therefore remains important for learned inverse mappings as well.

The transcript is of interest as a research dialogue about selecting the target of an inverse problem. Its value lies in the consistent transition from discussion of individual methods to the question of what an experiment allows us to learn about the medium. This question should remain central.

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